BOOK 5 – DEBT INVESTMENTS AND DERIVATIVE INVESTMENTS

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Best regards,

Doug Van Eaton, CFA
Vice President and Level 1 Manager
Schweser Study Program

READINGS AND LEARNING OUTCOME STATEMENTS

READINGS

The following material is a review of the Debt Investments and Derivative Investments principles designed to address the learning outcome statements set forth by CFA Institute.

STUDY SESSION 14

Reading Assignments

Fixed Income Analysis for the Chartered Financial Analyst® Program, 2nd edition, Frank J. Fabozzi (Frank J. Fabozzi Associates, 2004)

STUDY SESSION 15

Reading Assignments

Fixed Income Analysis for the Chartered Financial Analyst® Program, 2nd edition, Frank J. Fabozzi (Frank J. Fabozzi Associates, 2004)

STUDY SESSION 16

Reading Assignments

Analysis of Derivatives for the CFA® Program, Don Chance (AIMR, 2003)

Learning Outcome Statements (LOS)

The CFA Institute Learning Outcome Statements are listed below. These are repeated in each topic review; however, the order may have been changed in order to get a better fit with the flow of the review.

Study Session 14

The topical coverage corresponds with the following CFA Institute assigned reading:

62. **"Features of Debt Securities"**

The candidate should be able to

a. explain the purposes of a bond's indenture, and describe affirmative and negative covenants. (page 4)

b. describe the basic features of a bond (e.g., maturity, par value, coupon rate, provisions for redeeming bonds, currency denomination, options granted to the issuer or investor), the various coupon rate structures (e.g., zero-coupon bonds, step-up notes, deferred coupon bonds, floating-rate securities), the structure of floating-rate securities (i.e., the coupon formula, caps and floors), and define accrued interest, full price, and clean price. (page 4)

c. explain the provisions for early retirement of debt, including call and refunding provisions, prepayment options, and sinking fund provisions, differentiate between a regular redemption price and a special redemption price and explain the importance of options embedded in a bond issue, and indicate whether such options benefit the issuer or the bondholder. (page 7)

d. describe methods used by institutional investors in the bond market to finance the purchase of a security (i.e., margin buying and repurchase agreements). (page 9)

The topical coverage corresponds with the following CFA Institute assigned reading:

63. **"Risks Associated with Investing in Bonds"**

The candidate should be able to

a. explain the risks associated with investing in bonds (e.g., interest rate risk, yield curve risk, call and prepayment risk, reinvestment risk, credit risk, liquidity risk, exchange-rate risk, inflation risk, volatility risk, and event risk). (page 17)

b. identify the relationship among a bond's coupon rate, the yield required by the market, and the bond's price relative to par value (i.e., discount, premium, or equal to par). (page 18)

c. explain how features of a bond (e.g., maturity, coupon, and embedded options) affect the bond's interest rate risk. (page 19)

d. identify the relationship among the price of a callable bond, the price of an optionfree bond, and the price of the embedded call option. (page 20)

e. explain the interest rate risk of a floating-rate security and why such a security's price may differ from par value. (page 21)

f. compute and interpret the duration of a bond, given the bond's change in price when interest rates change, the approximate percentage price change of a bond, given the bond's duration, and the approximate new price of a bond, given the bond's duration and new yield level, explain why duration does not account for yield curve risk for a portfolio of bonds, and explain how the yield level impacts the interest rate risk of a bond. (page 21)

g. explain the disadvantages of a callable or prepayable security to an investor. (page 24)

h. identify the factors that affect the reinvestment risk of a security and explain why prepayable amortizing securities expose investors to greater reinvestment risk than nonamortizing securities. (page 24)
i. describe the various forms of credit risk (i.e., default risk, credit spread risk, downgrade risk) and describe the meaning and role of credit ratings. (page 25)
j. explain why liquidity risk might be important to investors even if they expect to hold a security to the maturity date. (page 26)
k. describe the exchange rate risk an investor faces when a bond makes payments in a foreign currency. (page 26)
l. describe inflation risk and explain why it exists. (page 26)
m. explain how yield volatility affects the price of a bond with an embedded option and how changes in volatility affect the value of a callable bond and a putable bond. (page 26)
n. describe the various forms of event risk (e.g., natural catastrophe, corporate takeover/restructuring and regulatory risk) and the components of sovereign risk. (page 27)

The topical coverage corresponds with the following CFA Institute assigned reading:

64. "Overview of Bond Sectors and Instruments"

The candidate should be able to

a. describe the different types of international bonds (e.g., Eurobonds, global bonds, sovereign debt). (page 36)
b. describe the types of securities issued by the U.S. Department of the Treasury (e.g. bills, notes, bonds, and inflation protection securities), differentiate between on-the-run and off-the-run Treasury securities, discuss how stripped Treasury securities are created, and distinguish between coupon strips and principal strips. (page 36)
c. describe a mortgage-backed security, and explain the cash flows for a mortgage-backed security, define prepayments, and explain prepayment risk. (page 38)
d. describe the types and characteristics of securities issued by federal agencies (including mortgage passthroughs and collateralized mortgage obligations). (page 39)
e. state the motivation for creating a collateralized mortgage obligation, describe the types of securities issued by municipalities in the United States, and distinguish between tax-backed debt and revenue bonds. (page 40)
f. describe insured bonds and prerefunded bonds. (page 42)
g. summarize the bankruptcy process and bondholder rights, explain the factors considered by rating agencies in assigning a credit rating to a corporate debt instrument, and describe secured debt, unsecured debt, and credit enhancements for corporate bonds. (page 42)
h. distinguish between a corporate bond and a medium-term note. (page 44)
i. describe a structured note, explain the motivation for their issuance by corporations, describe commercial paper, and distinguish between directly-placed paper and dealer-placed paper, and describe the salient features, uses and limitations of bank obligations (negotiable CDs and bankers acceptances). (page 44)
j. define an asset-backed security, describe the role of a special purpose vehicle in an asset-backed securities transaction, state the motivation for a corporation to issue an asset-backed security, and describe the types of external credit enhancements for asset-backed securities. (page 46)
k. describe collateralized debt obligations. (page 47)
l. contrast the structures of the primary and secondary markets in bonds. (page 47)

The topical coverage corresponds with the following CFA Institute assigned reading:

65. "Understanding Yield Spreads"

The candidate should be able to

a. identify the interest rate policy tools available to a central bank (such as the U.S. Federal Reserve or the European Central Bank). (page 55)
b. describe a yield curve and the different yield curve shapes observed and explain the basic theories of the term structure of interest rates (i.e., pure expectations theory, liquidity preference theory, and market segmentation theory) and describe the implications of each theory for the shape of the yield curve; explain the different types of yield spread measures (e.g., absolute yield spread, relative yield spread, yield ratio), and compute yield spread measures given the yields for two securities. (page 56)
c. explain why investors may find a relative yield spread to be a better measure of yield spread than the absolute yield spread, distinguish between an intermarket and intramarket sector spread, and describe a credit spread and discuss the suggested relationship between credit spreads and the economic well being of the economy. (page 60)
d. identify how embedded options affect yield spreads. (page 61)
e. explain how the liquidity of an issue affects its yield spread relative to Treasury securities and relative to other issues that are comparable in all other ways except for liquidity and describe the relationships that are argued to exist among the size of an issue, liquidity, and yield spread. (page 61)
f. compute the after-tax yield of a taxable security and the tax-equivalent yield of a tax-exempt security. (page 61)
g. define LIBOR and why it is an important measure to funded investors who borrow short-term. (page 62)

STUDY SESSION 15

The topical coverage corresponds with the following CFA Institute assigned reading:

66. "Introduction to the Valuation of Debt Securities"

The candidate should be able to

a. describe the fundamental principles of bond valuation. (page 70)
b. identify the types of bonds for which estimating the expected cash flows is difficult, and explain the problems encountered when estimating the cash flows for these bonds. (page 70)
c. determine the appropriate interest rates for valuing a bond's cash flows, compute the value of a bond, given the expected annual or semiannual cash flows and the appropriate single (constant) or multiple (arbitrage-free rate curve) discount rates, explain how the value of a bond changes if the discount rate increases or decreases, and compute the change in value that is attributable to the rate change, and explain how the price of a bond changes as the bond approaches its maturity date, and compute the change in value that is attributable to the passage of time. (page 71)
d. compute the value of a zero-coupon bond, explain the arbitrage-free valuation approach and the market process that forces the price of a bond toward its arbitragefree value, determine whether a bond is undervalued or overvalued, given the bond's cash flows, appropriate spot rates or yield to maturity, and current market price, explain how a dealer could generate an arbitrage profit. (page 75)

The topical coverage corresponds with the following CFA Institute assigned reading:

67. "Yield Measures, Spot Rates, and Forward Rates"

The candidate should be able to

a. explain the sources of return from investing in a bond (i.e., coupon interest payments, capital gain/ loss, reinvestment income). (page 83)
b. compute the traditional yield measures for fixed-rate bonds (e.g., current yield, yield to maturity, yield to first call, yield to first par call date, yield to refunding, yield to put, yield to worst, cash flow yield) and explain the assumptions underlying traditional yield measures and the limitations of the traditional yield measures. (page 83)
c. explain the importance of reinvestment income in generating the yield computed at the time of purchase, and calculate the amount of income required to generate that yield and discuss the factors that affect reinvestment risk. (page 89)
d. compute the bond equivalent yield of an annual-pay bond, and compute the annualpay yield of a semiannual-pay bond. (page 90)
e. compute the theoretical Treasury spot rate curve, using the method of bootstrapping and given the Treasury par yield curve and compute the value of a bond using spot rates. (page 90)
f. explain the limitations of the nominal spread and differentiate among the nominal spread, the zero-volatility spread, and the option-adjusted spread for a bond with an embedded option, and explain the option cost. (page 94)
g. explain a forward rate, and compute the value of a bond using forward rates, explain and illustrate the relationship between short-term forward rates and spot rates, and compute spot rates given forward rates, and forward rates given spot rates. (page 96)

The topical coverage corresponds with the following CFA Institute assigned reading:

68. "Introduction to the Measurement of Interest Rate Risk"

The candidate should be able to

a. distinguish between the full valuation approach (the scenario analysis approach) and the duration/ convexity approach for measuring interest rate risk, and explain the advantage of using the full valuation approach. (page 108)
b. compute the interest rate risk exposure of a bond position or of a bond portfolio, given a change in interest rates. (page 108)
c. demonstrate the price volatility characteristics for option-free bonds when interest rates change (including the concept of "positive convexity"), the price volatility characteristics of callable bonds and prepayable securities when interest rates change (including the concept of "negative convexity"), and describe the price volatility characteristics of putable bonds. (page 110)
d. compute the effective duration of a bond, given information about how the bond's price will increase and decrease for given changes in interest rates, and compute the approximate percentage price change for a bond, given the bond's effective duration and a specified change in yield. (page 112)
e. distinguish among the alternative definitions of duration (modified, effective or option-adjusted, and Macaulay), explain why effective duration is the most appropriate measure of interest rate risk for bonds with embedded options, describe why duration is best interpreted as a measure of a bond's or portfolio's sensitivity to changes in interest rates, compute the duration of a portfolio, given the duration of the bonds comprising the portfolio, and discuss the limitations of portfolio duration. (page 114)
f. discuss the convexity measure of a bond and estimate a bond's percentage price change, given the bond's duration and convexity and a specified change in interest rates. (page 116)
g. differentiate between modified convexity and effective convexity. (page 118)
h. compute the price value of a basis point (PVBP), and explain its relationship to duration. (page 118)

Study Session 16

The topical coverage corresponds with the following CFA Institute assigned reading:

69. "Derivative Markets and Instruments"

The candidate should be able to

a. define a derivative and differentiate between exchange-traded and over-the-counter derivatives. (page 128)
b. define a forward commitment, identify the types of forward commitments, and describe the basic characteristics of forward contracts, futures contracts, and swaps. (page 128)
c. define a contingent claim and identify the types of contingent claims. (page 129)
d. describe the basic characteristics of options, and distinguish between an option to buy (call) and an option to sell (put). (page 129)
e. discuss the purposes and criticisms of derivative markets. (page 129)
f. explain the concept of arbitrage and the role it plays in determining prices and in promoting market efficiency. (page 129)

The topical coverage corresponds with the following CFA Institute assigned reading:

70. "Forward Markets and Contracts"

The candidate should be able to

a. discuss the differences between the positions held by the long and short parties to a forward contract in terms of delivery/settlement and default risk. (page 133)
b. describe the procedures for settling a forward contract at expiration, and discuss how a party to a forward contract can terminate a position prior to expiration as well as how credit risk is affected by the way in which a position is terminated. (page 134)
c. differentiate between a dealer and an end user of a forward contract. (page 135)
d. describe the characteristics of equity forward contracts. (page 135)
e. describe the characteristics of forward contracts on zero-coupon and coupon bonds. (page 136)
f. explain the characteristics of the Eurodollar time deposit market, define LIBOR and Euribor, and describe the characteristics of forward rate agreements (FRAs). (page 137)
g. calculate and interpret the payment at expiration of an FRA, explain each of the component terms, and describe the characteristics of currency forward contracts. (page 137)

The topical coverage corresponds with the following CFA Institute assigned reading:

71. "Futures Markets and Contracts"

The candidate should be able to

a. identify the institutional features that distinguish futures contracts from forward contracts and describe the characteristics of futures contracts. (page 144)
b. differentiate between margin in the securities markets and margin in the futures markets. (page 145)
c. describe how a futures trade takes place. (page 145)
d. describe how a futures position may be closed out (i.e., offset) prior to expiration. (page 146)
e. define initial margin, maintenance margin, variation margin, and settlement price. (page 146)
f. describe the process of marking to market and compute the margin balance, given the previous day's balance and the new futures price. (page 146)
g. explain price limits, limit move, limit up, limit down, and locked limit. (page 147)

h. describe how a futures contract can be terminated by a close-out (i.e., offset) at expiration, delivery, an equivalent cash settlement, or an exchange-for-physicals. (page 148)
i. explain delivery options in futures contracts. (page 148)
j. distinguish among scalpers, day traders, and position traders. (page 148)
k. describe the characteristics of the following types of futures contracts: Treasury bill, Eurodollar, Treasury bond, stock index, and currency. (page 149)

The topical coverage corresponds with the following CFA Institute assigned reading:

72. **"Option Markets and Contracts"**

The candidate should be able to

a. identify the basic elements and describe the characteristics of option contracts. (page 156)
b. define European option, American option, moneyness, payoff, intrinsic value, and time value and differentiate between exchange-traded options and over-the-counter options. (page 157)
c. identify the different types of options in terms of the underlying instruments. (page 161)
d. compare and contrast interest rate options to forward rate agreements (FRAs). (page 161)
e. explain how option payoffs are determined, and show how interest rate option payoffs differ from the payoffs of other types of options. (page 162)
f. define interest rate caps and floors. (page 163)
g. identify the minimum and maximum values of European options and American options. (page 164)
h. explain how the lower bounds of European calls and puts are determined by constructing portfolio combinations that prevent arbitrage, and calculate an option's lower bound. (page 165)
i. determine the lowest prices of European and American calls and puts based on the rules for minimum values and lower bounds. (page 168)
j. describe how a portfolio (combination) of options establishes the relationship between options that differ only by exercise price. (page 168)
k. explain how option prices are affected by the time to expiration of the option. (page 169)
l. explain put-call parity for European options, given the payoffs on a fiduciary call and a protective put. (page 170)
m. explain the relationship between American options and European options in terms of the lower bounds on option prices and the possibility of early exercise. (page 171)
n. explain how cash flows on the underlying asset affect put-call parity and the lower bounds of option prices. (page 171)
o. identify the directional effect of an interest rate change on an option's price. (page 172)
p. describe the impact of a change in volatility on an option's price. (page 172)

The topical coverage corresponds with the following CFA Institute assigned reading:

73. **"Swap Markets and Contracts"**

The candidate should be able to

a. describe the characteristics of swap contracts and explain how swaps are terminated. (page 180)
b. define and give examples of currency swaps and calculate and interpret the payments on a currency swap. (page 181)
c. define and give an example of a plain vanilla interest rate swap and calculate and interpret the payments on an interest rate swap. (page 183)
d. define and give examples of equity swaps and calculate and interpret the payments on an equity swap. (page 185)

The topical coverage corresponds with the following CFA Institute assigned reading:

74. **"Risk Management Applications of Option Strategies"**

The candidate should be able to

a. determine the value at expiration, profit, maximum profit, maximum loss, breakeven underlying price at expiration, and general shape of the graph of the strategies of buying and selling calls and buying and selling puts, and explain each strategy's characteristics. (page 191)

b. determine the value at expiration, profit, maximum profit, maximum loss, breakeven underlying price at expiration, and general shape of the graph of the covered call strategy and the protective put strategy, and explain each strategy's characteristics. (page 194)

BONDS: THE BASICS

Study Session 14

THE BASICS

The following material offers students with a limited background in this topic area an overview of some of the basic concepts and definitions necessary to understand the material covered by the learning outcome statements in this study session. Those with adequate preparation in this topic area can proceed directly to the topic review for the first required reading in this study session.

BOND BASICS

The following two study sessions deal with debt (as opposed to equity) securities. In the U.S., corporations raise much more capital by issuing debt securities than by issuing equity securities, and the U.S. Treasury issues large amounts of debt securities on a regular basis. Debt securities are sometimes called fixed income securities, but you will soon realize that much of the complexity in valuing and determining the risk of "fixed income" securities is caused by the fact that the cash flows of many bonds are not really fixed.

Your understanding of the material may be improved with an explanation of the terms used when speaking about debt securities. Much of the terminology and some of the conventions for stating yields are based on historical practices.

Let's begin by looking at a simple fixed income (debt) security. Historically, a corporation that wanted to borrow money would issue bonds that were actual certificates. Each certificate (often with a nice engraved image of a factory or locomotive) had a *face value* (also called *par value*, *maturity value*, or *principal amount*) that was stated on the central portion of the certificate. A \$100,000 face value bond issued by the Reading Railroad Corporation on Jan. 15, 1955 with a maturity of 20 years would have a principal portion that would state something like, "The Reading Railroad Corporation will pay \$100,000 to the bearer on January 15, 1975." This represents the repayment in 20 years of the amount borrowed and is referred to as the *principal repayment* portion of the bond. The Reading Railroad is called the *issuer* of the security (bond) and is also the borrower of the money. Such a bond was called a *bearer bond.* If ownership is registered and a lost or destroyed certificate can be replaced, it is called a *registered bond* instead.

The remainder of the certificate (the bond) consisted of the bond *coupons.* Each coupon was a separate promise to pay a given amount of interest on the \$100,000 principal amount, typically every six months from the issue date until the maturity date. If a \$100,000 bond were issued with a "6% coupon rate," each coupon would typically be for one-half of the 6% (annual) coupon rate, or 3% of \$100,000. Thus, a 20-year, \$100,000, 6% bond issued on Jan. 15, 1955 would have 40 coupons attached to it. These coupons would have dates of July 15 and January 15 each year from July 1955 through the maturity date of January 15, 1975, and each one would be a promise to pay \$3,000 on those dates. In order to sell a bond, all coupons with dates later than the date of sale must still be attached. Delivery of a bond that has all such coupons still attached is referred to as *good delivery.*

The buyer of a bond is the lender of the funds and is also referred to as the owner of the bond (debt security) or the *bondholder*. If the market rate of interest for a loan to the issuing company is equal to the coupon rate when the bond is issued, the bond will be worth (and trade at) its *face value*. Since the stream of payments (coupons every six months and the principal or face value on the maturity date) is fixed when the bond is issued, its value can change over time as the (market-determined) required yield (return for lending money for that time period to that issuer) changes. Simply put, when yields go up, the (present) value of those promised payments goes down and vice versa.

Bond certificates can be issued with any principal or face value, and for this reason, market values/prices are often expressed as a percentage of face value. "One bond" is usually used to refer to $1,000 of face value so that an order for 100 bonds would be for $100,000 of principal value. If the bonds are trading at 98 ½, the cost of 100 bonds will be $98,500. Since bonds were traded long before calculators were available, simplistic measures of yield, such as *current yield*, were used and are still with us today. Current yield is the annual interest divided by the bond price and is still given in the bond quotations for corporate bonds published in daily newspapers.

Rating agencies give bond purchasers an indication of the relative likelihood of *default* (default risk). An AAA (or triple-A) rated bond is judged to have the least risk of failing to make its promised interest and principal payments over its life. Bonds with greater risk of defaulting on promised payments have lower ratings such as AA (double-A), A, BBB, BB, etc.

So far, we have discussed bonds that are simply a legally binding promise of the issuer to make scheduled interest and principal payments. These bonds are called *debentures*. When the bond is backed by the pledge of specific collateral that the bondholder has a claim to if the issuer defaults on a promised payment, it is called a collateralized bond. If the collateral is real estate, the bond is referred to as *a mortgage bond*, and if the collateral is physical assets such as railroad cars, the bond is referred to as an *equipment trust certificate*. In general, bonds backed by specific collateral get higher bond ratings than debentures of the same issuer, and the "better" the collateral, the higher the bond rating.

The term "bond" typically refers to a debt security with 20 or more years to maturity, while the term "note" is a debt security issued with between 2 and 10 years to maturity. While this terminology is often used when referring to bonds (and notes) issued by the U.S. Treasury, these are not hard and fast rules. The term "bond" will often be used to refer to any debt security, while the term "note" may be used for any shorter-term promise to pay. Most often, debt securities issued with maturities of one year or less do not make interest payments prior to maturity, a single payment at maturity is the only promised payment. In the case of U.S. Treasury issues, such securities are referred to as *bills,* rather than *notes* or *bonds*.

Some bonds will have a provision that requires the issuer to pay off (*redeem*) a portion of a bond issue prior to maturity, according to a schedule. The provision to retire these bonds *"early"* is referred to as a *sinking fund provision*. If the issuer has the option to pay off the principal prior to maturity (and not honor remaining coupons), the bond has a *call feature* or provision and is said to be *callable*. The amount the issuer must pay to bondholders to redeem callable bonds early is referred to as the *call price* and this is typically greater than or equal to the face value of the bond.

One significant change over the years is that certificates are typically no longer issued. The ownership of debt securities is instead recorded by the *registrar*. Bonds where ownership is not evidenced by a physical certificate are said to be *book entry* bonds, and the promised payments are made when due by sending the payments to the registered owners or their designated bank or brokerage house. As you read the material in the following study sessions, you will learn about many variations on this basic description of bonds or debt securities. Coupons may not be fixed; principal may be paid at other than the original maturity date for a variety of reasons under different structures; and other options besides call options may be part of the agreement between bond buyers and bond issuers. It may help to remember that essentially any agreement made and agreed to by both the issuer and the security buyers can be legally entered into. The variety of coupon structures and other provisions is driven by the marketplace. Features that bondholders have a preference for will tend to decrease the borrowing

costs of issuers (lower yields), and features that benefit the issuer will require the issuer to promise higher interest payments as compensation to buyers.

As we mentioned at the beginning of this section, the amount of debt securities outstanding is huge, and the effort devoted to analyzing and managing portfolios of debt securities is large as well. Evaluating the risk and potential return from debt is complicated, not only by the great variety of debt securities available, but by the number of issuers as well. This presents many opportunities for analysts who understand these securities and the markets they trade in.

The following is a review of the Debt Investments: Basic Concepts principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

Features of Debt Securities

Study Session 14

Exam Focus

Fixed income securities, historically, were promises to pay a stream of semiannual payments for a given number of years and then repay the loan amount at the maturity date. The contract between the borrower and the lender (the indenture) can really be designed to have any payment stream or pattern that the parties agree to. Types of contracts that are used frequently have specific names, and there is no shortage of those (for you to learn) here. You should pay special attention to how the periodic payments are determined (fixed, floating, and variants of these) and to how/when the principal is repaid (calls, puts, sinking funds, amortization, and prepayments). These features all affect the value of the securities and will come up again when you learn how to value these securities and compare their risks, both at Level 1 and Level 2.

LOS 62.a: Explain the purposes of a bond's indenture, and describe affirmative and negative covenants.

The contract that specifies all the rights and obligations of the issuer and the owners of a fixed income security is called the **bond indenture**. The indenture defines the obligations of and restrictions on the borrower and forms the basis for all future transactions between the bondholder and the issuer. These contract provisions are known as *covenants* and include both *negative covenants* (prohibitions on the borrower) and *affirmative covenants* (actions that the borrower promises to perform) sections.

Negative covenants include restrictions on asset sales (the company can't sell assets that have been pledged as collateral), negative pledge of collateral (the company can't claim that the same assets back several debt issues simultaneously), and restrictions on additional borrowings (the company can't borrow additional money unless certain financial conditions are met).

Affirmative covenants include the maintenance of certain financial ratios and the timely payment of principal and interest. For example, the borrower might promise to maintain the company's current ratio at a value of two or higher. If this value of the current ratio is not maintained, then the bonds could be considered to be in (technical) default.

LOS 62.b: Describe the basic features of a bond (e.g., maturity, par value, coupon rate, provisions for redeeming bonds, currency denomination, options granted to the issuer or investor), the various coupon rate structures (e.g., zero-coupon bonds, step-up notes, deferred coupon bonds, floating-rate securities), the structure of floating-rate securities (i.e., the coupon formula, caps and floors), and define accrued interest, full price, and clean price.

A "straight" (option-free) bond is the simplest case. Consider a Treasury bond that has a 6 percent **coupon** and **matures** five years from today in the amount of $1,000. This bond is a promise by the **issuer** (the U.S. Treasury) to pay 6 percent of the $1,000 **par value** (i.e., $60) each year for five years and to repay the $1,000 five years from today.

With Treasury bonds and almost all U.S. corporate bonds, the annual interest is paid in two semiannual installments. Therefore, this bond will make nine coupon payments (one every six months) of $30 and a final

payment of $1,030 (the par value plus the final coupon payment) at the end of five years. This stream of payments is fixed when the bonds are issued and does not change over the life of the bond.

Note that each semiannual coupon is one-half the coupon rate (which is always expressed as an annual rate) times the par value which is sometimes called the *face value* or *maturity value*. An 8 percent Treasury note with a face value of $100,000 will make a coupon payment of $4,000 every six months and a final payment of $104,000 at maturity.

A U.S. Treasury bond is denominated (of course) in U.S. dollars. Bonds can be issued in other currencies as well. The **currency denomination** of a bond issued by the Mexican government will likely be Mexican pesos. Bonds can be issued that promise to make payments in any currency.

The Treasury bond we described above pays only interest until maturity, at which time the entire par or face value is repaid. This repayment structure is referred to as a "bullet bond" or "bullet maturity." Alternatively, the bond terms may specify that the principal be repaid through a series of payments over time or all at once prior to maturity, at the option of either the bondholder or the issuer.

Amortizing securities make periodic *principal and interest* payments. A conventional mortgage is an example of an amortizing loan; the payments are all equal, and each payment consists of the periodic interest payment and the repayment of a portion of the original principal. For a fully amortizing loan, the final (level) payment at maturity retires the last remaining principal on the loan (e.g., a typical automobile loan).

Options Granted to Issuer or Investor

The following are examples of *embedded options*, embedded in the sense that they are an integral part of the bond contract and are not a separate security. Some embedded options are exercisable at the option of the issuer of the bond, and some are exercisable at the option of the purchaser of the bond. We only introduce these terms here; they are covered in more detail later in this study session and elsewhere in the curriculum.

Security owner options. In the following cases, the option embedded in the fixed-income security is an option granted to the security holder (lender) and gives additional value to the security, compared to an otherwise-identical straight (option-free) security.

1. *A conversion option* grants the holder of a bond the right to convert the bond into a fixed number of common shares of the issuer. This choice/option has value for the bondholder. An exchange option is similar but allows conversion of the bond into a security other than the common stock of the issuer.

2. *Put provisions* give bondholders the right to sell (put) the bond to the issuer at a specified price prior to maturity. The put price is generally par if the bonds were originally issued at or close to par. If interest rates have risen and/or the creditworthiness of the issuer has deteriorated so that the market price of such bonds has fallen below par, the bondholder may choose to exercise the put option and require the issuer to redeem the bonds at the put price.

3. *Floors* set a minimum on the coupon rate for a floating-rate bond, a bond with a coupon rate that changes each period based on a reference rate, usually a short-term rate such as LIBOR or the T-bill rate.

Security issuer options. In these cases, the embedded option is exercisable at the option of the issuer of the fixed income security. Securities where the issuer chooses whether to exercise the embedded option will be priced less (or with a higher coupon) than otherwise identical securities that do not contain such an option.

1. *Call provisions* give the bond issuer the right to redeem (pay off) the issue prior to maturity. The details of a call feature are covered later in this topic review.

2. *Prepayment options* are included in many amortizing securities, such as those backed by mortgages or car loans. A prepayment option gives the borrower/issuer the right to prepay the loan balance prior to maturity, in whole or in part, without penalty. Loans may be prepaid for a variety of reasons, such as the refinancing of a mortgage due to a drop in interest rates or the sale of a home prior to its loan maturity date.

3. *Accelerated sinking fund provisions* are embedded options held by the issuer that allow the issuer to (annually) retire a larger proportion of the issue than is required by the sinking fund provision, up to a specified limit.

4. *Caps* set a maximum on the coupon rate for a floating-rate bond, a bond with a coupon rate that changes each period based on a reference rate, usually a short-term rate such as LIBOR or the T-bill rate.

Coupon Rate Structures: Zero-Coupon Bonds, Step-Up Notes, Deferred Coupon Bonds

Zero-coupon bonds are bonds that do not pay periodic interest. They pay the par value at maturity and the interest results from the fact that zero-coupon bonds are initially sold at a price below par value (i.e., they are sold at a significant *discount to par value*). Sometimes we will call debt securities with no explicit interest payments *pure discount securities.*

Accrual bonds are similar to zero-coupon bonds in that they make no periodic interest payments prior to maturity, but different in that they are sold originally at (or close to) par value. There is a stated coupon rate, but the coupon interest accrues (builds up) at a compound rate until maturity. At maturity the par value plus all of the interest that has accrued over the life of the bond is paid.

Step-up notes have coupon rates that increase over time at a specified rate. The increase may take place once or more during the life of the issue.

Deferred-coupon bonds carry coupons, but the initial coupon payments are deferred for some period. The coupon payments accrue, at a compound rate, over the deferral period and are paid as a lump sum at the end of that period. After the initial deferment period has passed, these bonds pay regular coupon interest for the rest of the life of the issue (to maturity).

Floating-Rate Securities

Floating-rate securities are bonds for which the coupon interest payments over the life of the security vary based on a specified interest rate or index. For example, if market interest rates are moving up, the coupons on straight floaters will rise as well. In essence, these bonds have coupons that are reset periodically (normally every 3, 6, or 12 months) based on prevailing market interest rates.

The most common procedure for setting the coupon rates on floating-rate securities is one which starts with a *reference rate* (such as the rate on certain U.S. Treasury securities or the London Interbank Offered Rate [LIBOR]) and then adds or subtracts a stated *margin* to or from that reference rate. The quoted margin may also vary over time according to a schedule that is stated in the indenture. The schedule is often referred to as the *coupon formula.* Thus, to find the new coupon rate, you would use the following coupon formula:

new coupon rate = reference rate +/– quoted margin

Just as with a fixed-coupon bond, a semiannual coupon payment will be one-half the (annual) coupon *rate.*

Caps and floors. The parties to the bond contract can limit their exposure to extreme fluctuations in the reference rate by placing upper and lower limits on the coupon rate. The upper limit, which is called a cap, puts a *maximum* on the interest rate paid by the borrower/issuer. The lower limit, called a floor, puts a minimum on the periodic coupon interest payments received by the lender/security owner. When both limits are present simultaneously, the combination is called a *collar*.

Consider a floating rate security (floater) with a coupon rate at issuance of 5%, a 7% cap, and a 3% floor. If the coupon rate (reference rate plus the margin) rises above 7%, the borrower will pay (lender will receive) only 7% for as long as the coupon rate according to the formula remains at or above 7%. If the coupon rate falls below 3%, the borrower will pay 3% for as long as the coupon rate according to the formula remains at or below 3%.

Accrued Interest, Full Price, and Clean Price

When a *bond trades between coupon dates*, the seller is entitled to receive any interest earned from the previous coupon date through the date of the sale. This is known as **accrued interest** and is an amount that's payable by the buyer (new owner) of the bond. The new owner of the bond will receive *all of the next coupon payment* and will then recover any accrued interest paid on the date of purchase. The accrued interest is calculated as the fraction of the coupon period that has passed times the coupon.

In the U.S., the convention is for the bond buyer to pay any accrued interest to the bond seller. The amount that the buyer pays to the seller is the agreed-upon price of the bond (the **clean price**) *plus any accrued interest.* In the U.S., bonds trade with the next coupon attached, which is termed *cum coupon.* A bond traded without the right to the next coupon is said to be trading *ex-coupon.* The total amount paid, including accrued interest, is known as the **full (or dirty) price** of the bond. The full price = clean price + accrued interest.

If the issuer of the bond is in default (i.e., has not made periodic obligatory coupon payments), the bond will trade without accrued interest, and it is said to be trading *flat.*

LOS 62.c: Explain the provisions for early retirement of debt, including call and refunding provisions, prepayment options, and sinking fund provisions, differentiate between a regular redemption price and a special redemption price and explain the importance of options embedded in a bond issue, and indicate whether such options benefit the issuer or the bondholder.

Prepayment options give the issuer/borrower the right to accelerate the principal repayment on a loan. These options are present in mortgages and other *amortizing loans,* such as automobile loans. Amortizing loans require a series of equal payments that cover the periodic interest and reduce the outstanding principal each time a payment is made. When a person gets a home mortgage or an automobile loan, she often has the right to prepay it at any time, in whole or in part. If the borrower sells the home or auto, she is required to pay the loan off in full. The significance of a prepayment option to an investor in a mortgage or mortgage-backed security is that there is additional uncertainty about the cash flows to be received compared to a security that does not permit prepayment.

Call provisions give the issuer the right (but not the obligation) *to retire all or a part of an issue prior to maturity.* If the bonds are "called," the bondholders have no choice but to surrender their bonds for the call price because the bonds quit paying interest when they are called. Call features give the issuer the opportunity to replace higher-than-market coupon bonds with lower-coupon issues.

Typically, there is a period of years after issuance during which the bonds cannot be called. This is termed the period of *call protection* because the bondholder is protected from a call over this period. After the period (if any) of call protection has passed, the bonds are referred to as *currently callable.*

There may be several call dates specified in the indenture, each with a lower call price. Customarily, when a bond is called on the first permissible call date, the call price is above the par value. If the bonds are not called entirely

or not called at all, the call price declines over time according to a schedule. For example, a call schedule may specify that a 20-year bond can be called after 5 years at a price of 110 (110 percent of par), with the call price declining to 105 after 10 years and 100 in the fifteenth year.

Nonrefundable bonds prohibit the call of an issue using the proceeds from a lower coupon bond issue. Thus, a bond may be callable but not refundable. A bond that is *noncallable* has absolute protection against a call prior to maturity. In contrast, a callable but *nonrefundable* bond can be called for any reason other than refunding.

When bonds are called through a call option or through the provisions of a sinking fund, the bonds are said to be **redeemed.** If a lower coupon issue is sold to provide the funds to call the bonds, the bonds are said to be **refunded.**

Sinking fund provisions provide for the repayment of principal through a series of payments over the life of the issue. For example, a 20-year issue with a face amount of $300 million may require that the issuer retire $20 million of the principal every year beginning in the sixth year. This can be accomplished in one of two ways—*cash or delivery:*

- *Cash payment.* The issuer may deposit the required cash amount annually with the issue's trustee who will then retire the applicable proportion of bonds (1/15 in this example) by using a selection method such as a lottery. The bonds selected by the trustee are typically retired at par.
- *Delivery of securities.* The issuer may purchase bonds with a total par value equal to the amount that is to be retired in that year in the market and deliver them to the trustee who will retire them.

If the bonds are trading below par value, delivery of bonds purchased in the open market is the less expensive alternative. If the bonds are trading above the par value, delivering cash to the trustee to retire the bonds at par is the less expensive way to satisfy the sinking fund requirement.

Some indentures grant an option to the issuer to retire more than the required annual sinking fund amount. This feature is called an **accelerated sinking fund provision.** (*See Exam Flashbacks #1 and #2.*)

Regular and Special Redemption Prices

When bonds are redeemed under the call provisions specified in the bond indenture, these are known as regular redemptions, and the call prices are referred to as **regular redemption prices.** However, when bonds are redeemed to comply with a sinking fund provision or because of a property sale mandated by government authority, the redemption prices (typically par value) are referred to as **special redemption prices.** Asset sales may be forced by a regulatory authority (e.g., the forced divestiture of an operating division by antitrust authorities or through a governmental unit's right of eminent domain). Examples of sales forced through the government's right of eminent domain would be a forced sale of privately held land for erection of electric utility lines or for construction of a freeway.

Options Embedded in a Bond Issue

As noted earlier, the presence of **embedded options** must be taken into account in the pricing/valuation of bonds.

Recall that the following embedded options favor the *issuer/borrower:* (1) the right to call the issue, (2) an accelerated sinking fund provision, (3) a prepayment option, and (4) the cap on the floating coupon rate that limits the amount of interest payable by the borrower/issuer.

The following embedded options favor the *bondholders:* (1) conversion provisions, (2) a floor that guarantees a minimum interest payment to the bondholder, and (3) a put option. (*See Exam Flashback #3.*)

LOS 62.d: Describe methods used by institutional investors in the bond market to finance the purchase of a security (i.e., margin buying and repurchase agreements).

A repurchase (repo) agreement is an arrangement by which an institution sells a security with a commitment to buy it back at a later date at a specified (higher) price. The *repurchase price* is greater than the selling price and accounts for the interest charged by the buyer, who is, in effect, lending funds to the seller. The interest rate implied by the two prices is called the *repo rate*, which is the annualized percentage difference between the two prices. A repurchase agreement for one day is called an *overnight repo*, and an agreement covering a longer period is called a *term repo*. The interest cost of a *repo* is customarily less than the rate a bank or brokerage would charge on a margin loan.

Margin buying involves borrowing funds from a broker or a bank to purchase securities where the securities themselves are the collateral for the margin loan. The margin amount (percentage of the bonds' value) is regulated by the Federal Reserve in the U.S., under the Securities and Exchange Act of 1934.

Most bond-dealer financing is achieved through repurchase agreements rather than through margin loans. Repurchase agreements are not regulated by the Federal Reserve, and the collateral position of the lender/buyer in a repo is better in the event of bankruptcy of the dealer, since the security is owned by the 'lender.' The lender has only the obligation to sell it back at the price specified in the repurchase agreement, rather than simply having a claim against the assets of the dealer for the margin loan amount.

KEY CONCEPTS

1. The obligations, rights, and any options the issuer or owner of a bond may have are contained in the bond indenture. The specific conditions of the obligation are *covenants*. Affirmative covenants specify acts that the borrower must perform, and negative covenants prohibit the borrower from performing certain acts.
2. Bonds have the following features:
   - Maturity—the term of the loan agreement.
   - Par value—the principal amount of the fixed income security that the borrower promises to pay the lender on or before the bond expires at maturity.
   - Coupon—the rate that determines the periodic interest to be paid on the principal amount. Interest can be paid annually or semiannually, depending on the terms. Coupons may be fixed or variable.
3. Types of fixed-income securities:
   - Zero-coupon bonds pay no periodic interest and are sold at a discount to par value.
   - Accrual bonds pay compounded interest, but the cash payment is deferred until maturity.
   - Step-up notes have a coupon rate that increases over time according to a specified schedule.
   - Deferred coupon bonds initially make no coupon payments (they are deferred for a period of time). At the end of the deferral period, the accrued (compound) interest is paid, and the bonds then make regular coupon payments.
4. A floating (variable) rate bond has a coupon formula that is based on a reference rate (usually LIBOR) and a quoted margin. Caps are a maximum on the coupon rate that the issuer must pay, and a floor is a minimum on the coupon rate that the bondholder will receive.
5. Accrued interest is the interest earned since the last coupon payment date and is paid by a bond buyer to a bond seller. Clean price is the quoted price of the bond without accrued interest, and full price refers to the quoted price plus any accrued interest.
6. Bond payoff provisions:
   - Amortizing securities make periodic payments that include both interest and principal payments so that the entire principal is paid off with the last payment.
   - A prepayment provision is present in some amortizing loans and allows the borrower to pay off principal at any time prior to maturity, in whole or in part.
   - Sinking fund provisions require that a part of a bond issue be retired at specified dates, usually annually.

- Call provisions enable the borrower to buy back the bonds from the investors (redeem them) at a price(s) specified in the bond indenture.
- Callable but nonrefundable bonds can be called, but their redemption cannot be funded by the simultaneous issuance of lower coupon bonds.

7. Regular redemption prices refer to prices specified for calls; special redemption prices (usually par value) are prices for bonds that are redeemed to satisfy sinking fund provisions or other provisions for early retirement, such as the forced sale of firm assets.
8. Embedded options that benefit the issuer reduce the bond's value to a bond purchaser; examples are call provisions and accelerated sinking fund provisions.
9. Embedded options that benefit bondholders increase the bond's value to a bond purchaser; examples are conversion options (the option of bondholders to convert their bonds into a certain number of shares of the bond issuer's common stock) and put options (the option of bondholders to return their bonds to the issuer at a preset price).
10. Institutions can finance secondary market bond purchases by margin buying (borrowing some of the purchase price, using the securities as collateral) or, most commonly, by repurchase (repo) agreements (an arrangement in which an institution sells a security with a promise to buy it back at an agreed-upon higher price at a specified later date).

Exam Flashbacks

Required CFA Institute disclaimer: Due to CFA® curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #103 from '01–'03 sample exams.

Omega Corp. has outstanding a $100 million, 9 percent coupon bond issue that is refund protected until July 1, 2010. This issue:
A. is noncallable.
B. is call protected until July 1, 2010.
C. currently may be redeemed with funds from general operations.
D. currently may be redeemed, but only if refunded by an issue with a lower cost.

Exam Flashback # 2

Source: Question #5 from '89 actual exam.

The refunding provision of an indenture allows bonds to be retired *unless*:
A. they are replaced with a new issue having a lower interest cost.
B. the remaining time to maturity is less than five years.
C. the stated time period in the indenture has not passed.
D. the stated time period in the indenture has passed.

Exam Flashback # 3

Source: Question #104 from '01–'03 sample exams.

The embedded option that is *least likely* to be a benefit to the issuer of debt securities is the:
A. floor on a floater.
B. right to call the issue.
C. accelerated sinking fund provision.
D. right of the underlying borrowers in a pool of loans to repay an amount in excess of the scheduled principal payment.

CONCEPT CHECKERS: FEATURES OF DEBT SECURITIES

1. A bond's indenture:
   A. pledges it as collateral.
   B. contains its covenants.
   C. is the same as a debenture.
   D. relates only to its interest and principal payments.

2. A bond has a par value of $5,000 and a coupon rate of 8.5 percent payable semiannually. What is the dollar amount of the semiannual coupon payment?
   A. $212.50.
   B. $238.33.
   C. $425.00.
   D. $476.66.

3. From the perspective of the bondholder, which of the following pairs of options would both ***add value*** to a straight (option-free) bond?
   A. Call option, conversion option.
   B. Accelerated sinking fund provision, put option.
   C. Put option, conversion option.
   D. Pre-payment option, exchange option.

4. A 10-year bond pays no interest for three years, then pays $229.25, followed by payments of $35 semiannually for 7 years and an additional $1,000 at maturity. This bond is a(n):
   A. accrual bond.
   B. zero-coupon bond.
   C. deferred coupon bond.
   D. step-up bond.

5. Consider a $1 million semiannual-pay floating-rate issue where the rate is reset on January 1 and July 1 each year. The reference rate is 6-month LIBOR, and the stated margin is +1.25%. If 6-month LIBOR is 6.5% on July 1, what will the next semiannual coupon be on this issue?
   A. $32,500.
   B. $38,750.
   C. $65,000.
   D. $77,500.

6. Which of the following statements is **TRUE** with regard to *floating-rate issues* that have *caps* and *floors*?
   A. A cap is an advantage to the bondholder, while a floor is an advantage to the issuer.
   B. A floor is an advantage to the bondholder, while a cap is an advantage to the issuer.
   C. A floor is a disadvantage to both the issuer and the bondholder, while a cap is an advantage to both the issuer and the bondholder.
   D. A floor is an advantage to both the issuer and the bondholder, while a cap is a disadvantage to both the issuer and the bondholder.

7. An investor paid a full price of $1,059.04 each for 100 bonds. The purchase was between coupon dates, and accrued interest was $23.54 per bond. What is each bond's *clean price*?
   A. $1,000.00.
   B. $1,059.04.
   C. $1,035.50.
   D. $1,082.58.

8. Which of the following statements is **TRUE** with regard to a *call provision*?
   A. A call provision is an advantage to the bondholder.
   B. A call provision will benefit the issuer in times of declining interest rates.
   C. A callable bond will trade at a higher price than an identical noncallable bond.
   D. A nonrefundable bond provides more protection to the bondholder than a noncallable bond.

9. Which of the following best describes the *maximum* price for a currently callable bond?
   A. Its par value.
   B. The call price.
   C. The present value of its par value.
   D. Its par value plus accrued interest.

Use the following information to answer Questions 10 and 11.

Consider an issue of $1,000,000 par value, 10-year, 6.5 percent coupon bonds issued on January 1, 2002. The bonds are callable and there is a sinking fund provision. The market rate for similar bonds is currently 5.70 percent. The main points of the prospectus are summarized as follows:

Call dates and prices:
- 2002 through 2006, 103.
- After Jan. 1, 2007, 102.

Additional information:
- Prior to January 1, 2006, the bonds are non-refundable.
- The sinking fund provision requires that the company redeem $100,000 of the principal amount each year. Bonds called under the terms of the sinking fund provision will be redeemed at par.
- The credit rating of the bonds is currently the same as at issuance.

10. Using only the above information, Gould should conclude that:
   A. investors will pay a premium for the call option.
   B. the bonds do not have call protection.
   C. the bonds were issued at and currently trade at a premium.
   D. given current rates, the bonds will likely be called and new bonds issued.

11. Which of the following statements is **TRUE**?
   A. An investor would benefit from having his or her bonds called under the provision of the sinking fund.
   B. An investor will receive a premium if the bond is redeemed prior to maturity.
   C. The bonds do not have an accelerated sinking fund provision.
   D. The issuer would likely deliver bonds to satisfy the sinking fund provision.

12. An investor buying bonds on margin:
   A. can achieve lower funding costs than one using repurchase agreements.
   B. must pay interest on a loan.
   C. is not restricted by government regulation of margin lending.
   D. actually "loans" the bonds to a bank or brokerage house.

13. Which of the following is **NOT** a provision for the early retirement of debt by the issuer?
   A. A conversion option.
   B. A call option.
   C. A prepayment option.
   D. A sinking fund.

14. A mortgage is typically **NOT**:
    A. a collateralized loan.
    B. subject to early retirement.
    C. an amortizing security.
    D. characterized by highly predictable cash flows.

ANSWERS – EXAM FLASHBACKS

1. **C** Refund protection does not mean that the bond cannot be called. In fact, most "refund protected" bonds are freely callable. "Nonrefundable" simply means that the bond cannot be refunded (replaced) by a new bond issue with a lower coupon rate. Nonrefundable bonds can be called (or redeemed) using cash from operations or a new equity issue.

2. **A** Nonrefundable bonds are typically freely callable. Although this question is very similar to the previous exam flashback, I wanted to include it because the language that is used in the question is slightly different. I'm trying to show that the same question can be asked in a variety of ways.

3. **A** The floor on a floating-rate bond prevents the coupon rate from being reset at any rate below the floor. Had the coupon been allowed to fall below the floor, this would have benefited the issuer. Hence, a floor can be detrimental to the issuer and beneficial to the buyer of the security. Response "D" is related to mortgage-backed securities. Here the "issuer" is the borrower or homeowner. In a falling interest rate environment, it can be highly beneficial for the borrower to refinance the mortgage at a lower interest rate.

ANSWERS – CONCEPT CHECKERS: FEATURES OF DEBT SECURITIES

1. **B** An indenture is the contract between the company and its bondholders and contains the bond's covenants.

2. **A** The annual interest is 8.5% of the $5,000 par value, or $425. Each semiannual payment is one-half of that, or $212.50.

3. **C** A put option, conversion option, and exchange option all have positive value to the bondholder. The other options favor the issuer and have a lower value than a straight bond.

4. **C** This pattern describes a deferred coupon bond. The first payment of 229.25 is the value of the accrued coupon payments for the first three years.

5. **B** The coupon rate is 6.5 + 1.25 = 7.75. The (semiannual) coupon payment equals (0.5)(0.0775)($1,000,000) = $38,750.

6. **B** A cap is a maximum on the coupon rate and is advantageous to the issuer. A floor is a minimum on the coupon rate and is therefore advantageous to the bondholder.

7. **C** The full price includes accrued interest, while the clean price does not. Therefore the clean price is 1,059.04 – 23.54 = $1,035.50.

8. **B** A call provision gives the bond issuer the right to call the bond at a price specified in the bond indenture. A bond issuer may want to call a bond if interest rates have decreased so that borrowing costs can be decreased by replacing the bond with a lower coupon issue.

9. **B** Whenever the price of the bond increases above the strike price stipulated on the call option, it will be optimal for the issuer to call the bond. So theoretically, the price of a currently callable bond should never rise above its call price.

10. **B** The bonds are callable in 2002, indicating that there is no period of call protection. We have no information about the pricing of the bonds at issuance. The company may not *refund* the bonds (i.e., they cannot call the bonds with the proceeds of a new debt offering at the currently lower market yield). The call option benefits the issuer, not the investor.

11. **C** The sinking fund provision does not provide for an acceleration of the sinking fund redemptions. With rates currently below the coupon rate, the bonds will be trading at a premium to par value. Thus a sinking fund call at par would not benefit a bondholder, and the issuer would likely deliver cash to the trustee to satisfy the sinking fund provision, rather than buying bonds to deliver to the trustee.

12. **B** Margin loans require the payment of interest, and the rate is typically higher than funding costs when repurchase agreements are used.

13. **A** A conversion option allows bondholders to exchange their bonds for common stock.

14. **D** A mortgage can typically be retired early in whole or in part (a prepayment option), and this makes the cash flows difficult to predict with any accuracy.

The following is a review of the Debt Investments: Basic Concepts principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

Risks Associated With Investing in Bonds

Study Session 14

Exam Focus

This topic review introduces various sources of risk that investors are exposed to when investing in fixed income securities. The key word here is introduces. The most important source of risk, interest rate risk, has its own full topic review in Study Session 15 and is more fully developed after the material on the valuation of fixed income securities. Prepayment risk has its own topic review at Level 2, and credit risk and reinvestment risk are revisited to a significant extent in other parts of the Level 1 curriculum. In this review we present some working definitions of the eleven risk measures enumerated in the LOS and identify the factors that will affect these risks. To avoid unnecessary repetition, some of the material is abbreviated here, but be assured that your understanding of this material will be complete by the time you work through this study session and the one that follows.

LOS 63.a: **Explain the risks associated with investing in bonds (e.g., interest rate risk, yield curve risk, call and prepayment risk, reinvestment risk, credit risk, liquidity risk, exchange-rate risk, inflation risk, volatility risk, and event risk).**

Interest rate risk refers to the effect of changes in the prevailing market rate of interest on bond values. When interest rates rise, bond values fall. This is the source of interest rate risk which is approximated by a measure called *duration*.

Yield curve risk arises from the possibility of changes in the shape of the yield curve (which shows the relation between bond yields and maturity). While duration is a useful measure of interest rate risk for equal changes in yield at every maturity (parallel changes in the yield curve), changes in the shape of the yield curve mean that yields change by different amounts for bonds with different maturities.

Call risk arises from the fact that when interest rates fall, a callable bond investor's principal may be returned and must be reinvested at the new lower rates. Certainly bonds that are not callable have no call risk, and call protection reduces call risk. When interest rates are more volatile, callable bonds have relatively more call risk because of an increased probability of yields falling to a level where the bonds will be called.

Prepayment risk is similar to call risk. Prepayments are principal repayments in excess of those required on amortizing loans, such as residential mortgages. If rates fall causing prepayments to increase, an investor must reinvest these prepayments at the new lower rate. Just as with call risk, an increase in interest rate volatility increases prepayment risk.

Reinvestment risk refers to the fact that when market rates fall, the cash flows (both interest and principal) from fixed-income securities must be reinvested at lower rates, reducing the returns an investor will earn. Note that reinvestment risk is related to call risk and prepayment risk. In both of these cases, it is the reinvestment of principal cash flows at lower rates than were expected that negatively impacts the investor. Coupon bonds that contain neither call nor prepayment provisions will also be subject to reinvestment risk, since the coupon interest payments must be reinvested as they are received.

Note that investors can be faced with a choice between reinvestment risk and price risk. A non-callable zero-coupon bond has no reinvestment risk over its life since there are no cash flows to reinvest, but a zero coupon

bond (as we will cover shortly) has more interest rate risk than a coupon bond of the same maturity. Therefore the coupon bond will have more reinvestment risk and less price risk.

Credit risk is the risk that the creditworthiness of a fixed-income security's issuer will deteriorate, increasing the required return and decreasing the security's value.

Liquidity risk has to do with the risk that the sale of a fixed-income security must be made at a price less than fair market value because of a lack of liquidity for a particular issue. Treasury bonds have excellent liquidity, so selling a few million dollars worth at the prevailing market price can be easily and quickly accomplished. At the other end of the liquidity spectrum, a valuable painting, collectible antique automobile, or unique and expensive home may be quite difficult to sell quickly at fair-market value. Since investors prefer more liquidity to less, a decrease in a security's liquidity will decrease its price, as the required yield will be higher.

Exchange-rate risk arises from the uncertainty about the value of foreign currency cash flows to an investor in terms of his home-country currency. While a U.S. Treasury bill may be considered quite low risk or even risk-free to a U.S.-based investor, the value of the T-bill to a European investor will be reduced by a depreciation of the U.S. dollar's value relative to the euro.

Inflation risk might be better described as *unexpected* inflation risk and even more descriptively as purchasing-power risk. While a $10,000 zero-coupon Treasury bond can provide a payment of $10,000 in the future with (almost) certainty, there is uncertainty about the amount of goods and services that $10,000 will buy at the future date. This uncertainty about the amount of goods and services that a security's cash flows will purchase is referred to here as inflation risk.

Volatility risk is present for fixed-income securities that have embedded options, such as call options, prepayment options, or put options. Changes in interest rate volatility affect the value of these options and thus affect the values of securities with embedded options.

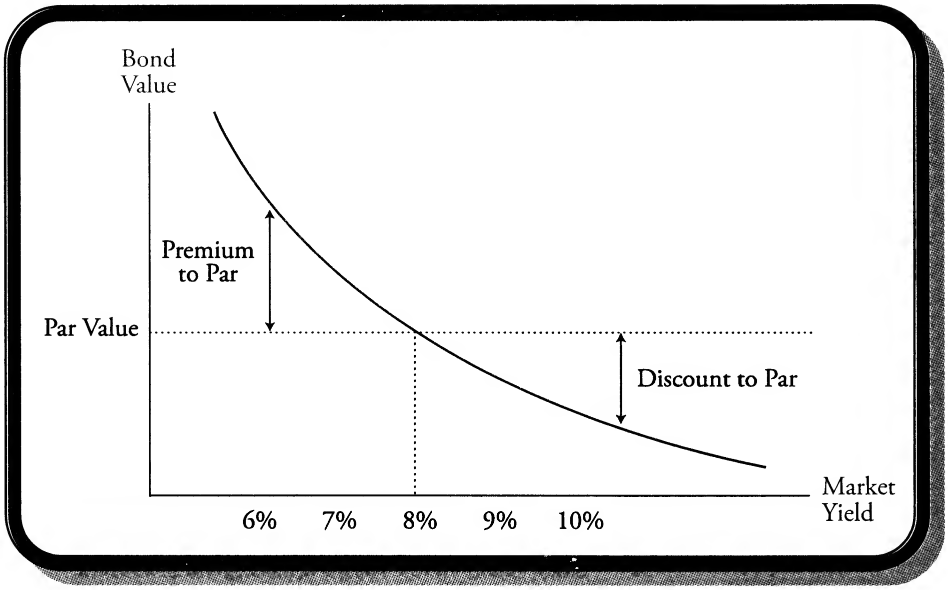
Event risk encompasses the risks outside the risks of financial markets, such as the risks posed by natural disasters and corporate takeovers.

LOS 63.b: Identify the relationship among a bond's coupon rate, the yield required by the market, and the bond's price relative to par value (i.e., discount, premium, or equal to par).

When the coupon rate on a bond is equal to its market yield, the bond will trade at its **par value**. When issued, the coupon rate on bonds is typically set at or near the prevailing market yield on similar bonds so that the bonds trade initially at or near their **par value.** If the yield required in the market for the bond subsequently rises, the price of the bond will fall and it will trade at a **discount** to (below) its par value. Conversely, if the required yield falls, the bond price will increase and the bond will trade at a **premium** to (above) its par value.

The relation is illustrated in Figure 1.

Figure 1: Market Yield vs. Bond Value for an 8% Coupon Bond



Professor's Note: This is a crucial concept and the reasons underlying this relation will be clear after you cover the material on bond valuation methods in the next study session.

LOS 63.c: Explain how features of a bond (e.g., maturity, coupon, and embedded options) affect the bond's interest rate risk.

Interest rate risk, as we are using it here, refers to the sensitivity of a bond's value to changes in market interest rates/yields. Remember that there is an inverse relationship between yield and bond prices—when yields increase, bond prices decrease. The term we use for the measure of interest rate risk is **duration,** which gives us a good approximation of a bond's change in price for a given change in yield.

Professor's Note: This is a very important concept. Notice that the terms bond price risk, interest rate risk, interest rate sensitivity, and duration are used interchangeably.

We introduce this concept by simply looking at how a bond's maturity and coupon affect its price sensitivity to interest rate changes. With respect to maturity, if two bonds are identical except for **maturity,** the one with the *longer maturity has the greater duration* since it will have a greater percentage change in value for a given change in yield. For two otherwise identical bonds, the one with the higher **coupon** rate has the lower duration. The price of the bond with the higher coupon rate will change less for a given change in yield than the price of the lower coupon bond will. (*See Exam Flashback #1.*)

The presence of **embedded options** also affects the sensitivity of a bond's value to interest rate changes (its duration). Prices of putable and callable bonds will react differently to changes in yield than the prices of straight (option-free) bonds will.

A call feature limits the upside price movement of a bond when interest rates decline; loosely speaking, the bond price will not rise above the call price. This leads to the conclusion that the value of a callable bond will be less sensitive to interest rate changes than an otherwise identical option-free bond.

A put feature limits the downside price movement of a bond when interest rates rise; loosely speaking, the bond price will not fall below the put price. This leads to the conclusion that the value of a putable bond will be less sensitive to interest rate changes than an otherwise identical option-free bond.

The relations we have developed so far are summarized in Figure 2.

Figure 2: Bond Characteristics and Interest Rate Risk

Characteristic	*Interest Rate Risk*	*Duration*
Maturity up	Interest rate risk up	Duration up
Coupon up	Interest rate risk down	Duration down
Add a call	Interest rate risk down	Duration down
Add a put	Interest rate risk down	Duration down

LOS 63.d: Identify the relationship among the price of a callable bond, the price of an option-free bond, and the price of the embedded call option.

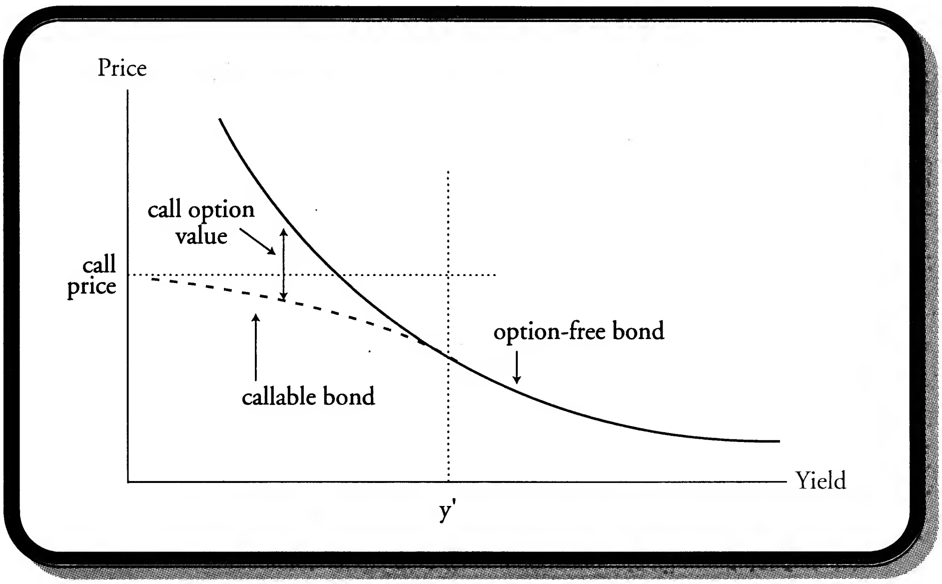
As we noted earlier, a call option favors the issuer and decreases the value of a callable bond relative to an otherwise identical option-free bond. The issuer owns the call. Essentially, when you purchase a callable bond, you have purchased an option-free bond but have "given" a call option to the issuer. The value of the callable bond is less than the value of an option-free bond in an amount equal to the value of the call option.

This relation can be shown as:

callable bond value = value of an option-free bond – value of the embedded call option

Figure 3 shows this relationship. The value of the call option is greater at lower yields so that as the yield falls, the difference in price between a straight bond and a callable bond increases.

Figure 3: Price-Yield Curves for Callable and Noncallable Bonds



LOS 63.e: Explain the interest rate risk of a floating-rate security and why such a security's price may differ from par value.

Recall that floating-rate securities have a coupon rate that 'floats,' in that it is periodically reset based on a market-determined reference rate. The objective of the resetting mechanism is to bring the coupon rate in line with the current market yield so the bond sells at or near its par value. This will make the price of a floating rate security much less sensitive to changes in market yields than a fixed-coupon bond of equal maturity. That's the point of a floating rate security: less interest rate risk.

Between coupon dates, there is a time lag between any change in market yield and a change in the coupon rate (which happens on the *reset* date). The longer the time period between the two dates, the greater the amount of potential bond price fluctuation. In general, we can say that the longer (shorter) the reset period, the greater (less) the interest rate risk of a floating rate security at any reset date.

The presence of a *cap* (maximum coupon rate) can increase the interest rate risk of a floating-rate security. If the reference rate increases enough that the cap rate is reached, further increases in market yields will decrease the floater's price. When the market yield is above its capped coupon rate, a floating-rate security will trade at a discount. To the extent that the cap fixes the coupon rate on the floater, its price sensitivity to changes in market yield will be increased. This is sometimes referred to as *cap risk*.

A floater's price can also differ from par due to the fact that the margin is fixed at issuance. Consider a firm that has issued floating rate debt with a coupon formula of LIBOR + 2 percent. This 2 percent margin should reflect the credit risk and liquidity risk of the security. If the firm's creditworthiness improves, the floater is less risky and will trade at a premium to par. Even if the firm's creditworthiness remains constant, a change in the market's required yield premium for the firm's risk level will cause the value of the floater to differ from par. (*See Exam Flashback #2.*)

LOS 63.f: Compute and interpret the duration of a bond, given the bond's change in price when interest rates change, the approximate percentage price change of a bond, given the bond's duration, and the approximate new price of a bond, given the bond's duration and new yield level, explain why duration does not account for yield curve risk for a portfolio of bonds, and explain how the yield level impacts the interest rate risk of a bond.

By now you know that duration is a measure of the price sensitivity of a security to changes in yield. Specifically, it can be interpreted as an approximation of the *percentage* change in the security price for a 1 percent change in yield. We can also interpret duration as the *ratio* of the percentage change in price to the change in yield in percent. This relation is:

$$\text{duration} = -\frac{\text{percentage change in bond price}}{\text{yield change in percent}}$$

When calculating the direction of the price change, remember that yields and prices are inversely related. If you are given a rate decrease, your result should indicate a price increase. Also note that the duration of a zero-coupon bond is approximately equal to its maturity, and the duration of a floater is equal to the time to the next reset date.

Let's consider some numerical examples.

Example 1: Approximate price change when yields increase

If a bond has a duration of 5 and the yield increases from 7 percent to 8 percent, **calculate** the approximate percentage change in the bond price.

Answer:

$-5 \times 1\% = -5\%$, or a 5 percent decrease in price. Since the yield increased, the price decreased.

Example 2: Approximate price change when yields decrease

A bond has a duration of 7.2. If the yield decreases from 8.3 percent to 7.9 percent, **calculate** the approximate percentage change in the bond price.

Answer:

$-7.2 \times (-0.4\%) = 2.88\%$, or an increase in price. Here the yield decreased and the price increased.

The "official" formula for what we just did (because duration is always expressed as a positive number and because of the negative relation between yield and price) is:

percentage price change = – duration × (yield change in %)

Now let's do it in reverse and calculate the duration from the change in yield and the *percentage* change in the bond's price.

Example 3: Calculating duration given a yield increase

If a bond's yield rises from 7 percent to 8 percent and its price falls 5 percent, **calculate** the duration.

Answer:

$$\text{duration} = -\frac{\text{percentage change in price}}{\text{change in yield}} = -\frac{-5.0\%}{+1.0\%} = 5$$

Example 4: Calculating duration given a yield decrease

If a bond's yield decreases by 0.1 percent and its price increases by 1.5 percent, **calculate** its duration.

Answer:

$$\text{duration} = -\frac{\text{percentage change in price}}{\text{change in yield}} = -\frac{1.5\%}{-0.1\%} = 15$$

Example 5: Calculating the new price of a bond

A bond is currently trading at $1,034.50, has a yield of 7.38 percent, and has a duration of 8.5. If the yield rises to 7.77 percent, **calculate** the new price of the bond.

Answer:

The change in yield is $7.77\% - 7.38\% = 0.39\%$.

The approximate price change is $-8.5 \times 0.39\% = -3.315\%$.

Since the yield *increased*, the price will decrease by this *percentage*.

The new price is $(1 - 0.03315) \times \$1,034.50 = \$1,000.21$.

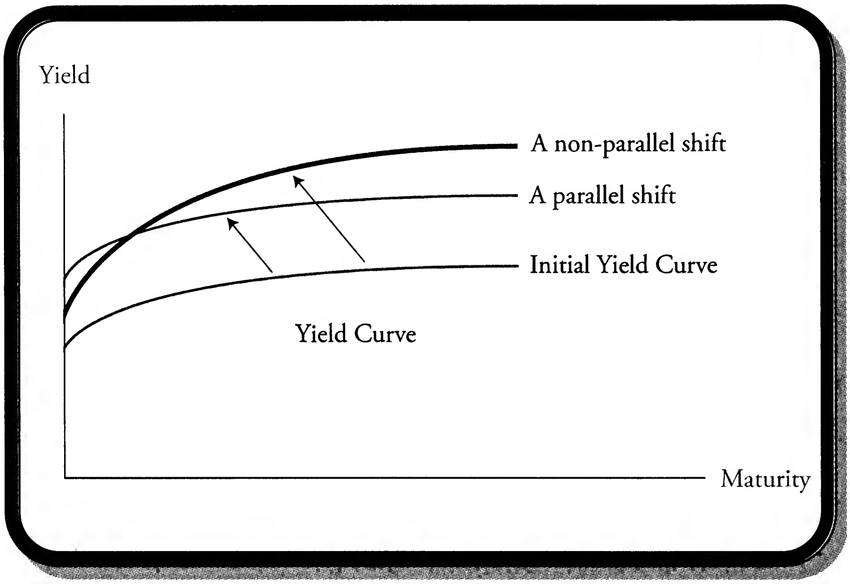
Duration and Yield Curve Risk for a Portfolio of Bonds

The duration for a portfolio of bonds has the same interpretation as for a single bond; it is the approximate percentage change in *portfolio* value for a 1% change in yields. Duration for a portfolio measures the sensitivity of a portfolio's value to a change in interest rates.

A graph of the relationship between maturity and yield is known as a *yield curve*. The yield curve can have any shape: upward sloping, downward sloping, flat, or some combination of these slopes. Changing yield curve shapes lead to **yield curve risk**, a risk of fixed income securities that is not captured by the duration measure.

In Figure 4 we illustrate two possible ways that the yield curve might change when market interest rates rise, a parallel change and a non-parallel change.

Figure 4: Yield Curve Shifts



The duration of a bond portfolio can be calculated from the individual bond durations and the proportions of the total portfolio value invested in each of the bonds. That is, the portfolio duration is a market-weighted average of the individual bond's durations. If the yields on all the bonds in the portfolio change by the same absolute percent amount, we term that a *parallel shift*. Portfolio duration is an approximation of the price sensitivity of a portfolio to parallel shifts of the yield curve.

For a non-parallel shift in the yield curve, the yields on different bonds in a portfolio can change by different amounts, and duration alone cannot capture the effect of a "yield change" on the value of the portfolio. This risk of decreases in portfolio value from changes in the shape of the yield curve (i.e., from non-parallel shifts in the yield curve) is termed *yield curve risk*. We will revisit this limitation of duration as a measure of interest rate risk in Study Session 15.

Yield Level and the Interest Rate Risk of a Bond

So far we have examined the effect of maturity, coupon, and embedded options on the interest rate risk (duration) of a bond. Now we examine one more factor that affects duration, the required yield in the market. It is a negative relation; the higher the yield, all else equal, the lower a bond's sensitivity to interest rate changes (i.e., the lower its duration).

We need to consider two examples:

1. If two option-free bonds are identical except for their market yields, the one with the lower yield will have more interest rate risk (greater duration).

2. If market yields increase for a given option-free bond, the interest rate risk of that bond (its duration) decreases.

Professor's Note: We have examined several factors that affect interest rate risk, but only maturity is positively related to interest rate risk (longer maturity, higher duration). To remember this, note that the words maturity and duration both have to do with time. The other factors, coupon rate, yield, and the presence of puts and calls, are all negatively related to interest rate risk (duration). Increasing coupons, higher yields, and "adding" options all decrease interest rate sensitivity (duration).

LOS 63.g: Explain the disadvantages of a callable or prepayable security to an investor.

Compared to an option-free bond, bonds with call provisions and securities with prepayment options offer a much less certain cash flow stream. This uncertainty about the timing of cash flows is one disadvantage of callable and prepayable securities.

A second disadvantage stems from the fact that the call of a bond and increased prepayments of amortizing securities are both more probable when interest rates have decreased. The disadvantage here is that more principal (all of the principal, in the case of a call) is returned when the opportunities for reinvestment of these principal repayments are less attractive. When rates are low, you get more principal back that must be reinvested at the new lower rates. When rates rise and opportunities for reinvestment are better, less principal is likely to be returned early.

A third disadvantage is that the potential price appreciation of callable and prepayable securities from decreases in market yields is less than that of option-free securities of like maturity. For a currently-callable bond, the call price puts an upper limit on the bond's price appreciation. While there is no equivalent price limit on a prepayable security, the effect of the prepayment option operates similarly to a call feature and reduces the appreciation potential of the securities in response to falling market yields.

Overall, the risks of early return of principal and the related problem of reinvesting the principal at lower rates are termed call risk and prepayment risk, respectively.

LOS 63.h: Identify the factors that affect the reinvestment risk of a security and explain why prepayable amortizing securities expose investors to greater reinvestment risk than nonamortizing securities.

As noted in our earlier discussion of reinvestment risk, cash flows prior to stated maturity from coupon interest payments, bond calls, principal payments on amortizing securities, and prepayments all subject security holders to reinvestment risk. Remember a lower coupon increases duration (interest rate risk) but decreases reinvestment risk compared to an otherwise identical higher coupon issue.

A security has *more* reinvestment risk when:

- The coupon is higher so that interest cash flows are higher.
- It has a call feature.
- It is an amortizing security.
- It contains a prepayment option.

(*See Exam Flashback #3.*)

Prepayable Amortizing Securities and Reinvestment Risk

As noted earlier, when interest rates decline there is an increased probability of the early return of principal for prepayable securities. The early return of principal increases the need to reinvest at lower prevailing rates. With prepayable securities, the uncertainty about the bondholder's return due to early return of principal and the prevailing reinvestment rates when it is returned (i.e., reinvestment risk) is greater.

LOS 63.i: Describe the various forms of credit risk (i.e., default risk, credit spread risk, downgrade risk) and describe the meaning and role of credit ratings.

A bond's *rating* is used to indicate its relative probability of default, which is the probability of its issuer not making timely interest and principal payments as promised in the bond indenture. A bond rating of AA is an indication that the expected probability of default over the life of the bond is less than that of an A-rated bond, which has a lower expected probability of default than a BBB ("triple B") rated bond, etc. We can say that lower-rated bonds have more **default risk,** the risk that a bond will fail to make promised/scheduled payments (either interest payments or principal payments). Since investors prefer less risk of default, a lower-rated issue must promise a higher yield to compensate investors for taking on a greater probability of default.

The difference between the yield on a Treasury security, which is assumed to be default risk free, and the yield on a similar maturity bond with a lower rating is termed the *credit spread.*

yield on a risky bond = yield on a default-free bond + default risk premium (credit spread)

Credit spread risk refers to the fact that the default risk premium required in the market for a given rating can increase, even while the yield on Treasury securities of similar maturity remains unchanged. An increase in this *credit spread* increases the required yield and decreases the price of a bond.

Downgrade risk is the risk that a credit rating agency will lower a bond's rating. The resulting increase in the yield required by investors will lead to a decrease in the price of the bond. A rating increase is termed an *upgrade* and will have the opposite effect, decreasing the required yield and increasing the price.

Rating agencies give bonds ratings which are meant to give bond purchasers an indication of the risk of default. While the ratings are primarily based on the financial strength of the company, different bonds of the same company can have slightly different ratings depending on differences in collateral or differences in the priority of the bondholders' claim (junior or subordinated bonds may get lower ratings than senior bonds). Bond ratings are not absolute measures of default risk, but rather give an indication of the relative probability of default across the range of companies and bonds.

For ratings given by Standard and Poor's Corporation, an AAA (triple-A) bond has been judged to have the least risk of failing to make its promised interest and principal payments (defaulting) over its life. Bonds with greater risk of defaulting on promised payments have lower ratings such as AA (double-A), A (single-A), BBB, BB, etc. U.S. Treasury securities and a small number of corporate bonds receive an AAA rating.

Pluses and minuses are used to indicate differences in default risk within categories, with AA+ a better rating than AA, which is better than AA-. Bonds rated AAA through BBB are considered 'investment grade' and bonds rated BB and below are considered speculative and sometimes termed 'junk bonds' or, more positively, 'high-yield bonds'. Bonds rated CCC, CC, and C are highly speculative and bonds rated D are currently in default. Moody's (Investor Services, Inc.), another prominent issuer of bond ratings, classifies bonds similarly but uses Aa1 as S&P uses AA+, Aa2 as AA, Aa3 as AA-, and so on. Bonds with lower ratings carry higher promised yields in the market because investors exposed to more default risk require a higher promised return to compensate them for bearing greater default risk.

LOS 63.j: Explain why liquidity risk might be important to investors even if they expect to hold a security to the maturity date.

We described liquidity earlier and noted that investors prefer more liquidity to less. This means that investors will require a higher yield for less liquid securities, other things equal. The difference between the price that dealers are willing to pay for a security (the bid) and the price at which dealers are willing to sell a security (the ask) is called the *bid-ask spread.* The bid-ask spread is an indication of the liquidity of the market for a security. If trading activity in a particular security declines, the bid-ask spread will widen (increase), and the issue is considered to be less liquid.

If investors are planning to sell a security prior to maturity, a decrease in liquidity will increase the bid-ask spread, lead to a lower sale price, and can decrease the returns on the position. Even if an investor plans to hold the security until maturity rather than trade it, poor liquidity can have adverse consequences stemming from the need to periodically assign current values to portfolio securities. This periodic valuation is referred to as *marking to market.* When a security has little liquidity, the variation in dealers' bid prices or the absence of dealer bids altogether makes valuation difficult and may require that a valuation model or pricing service be used to establish current value. If this value is low, institutional investors may be negatively impacted in two situations.

1. Institutional investors may need to mark their holdings to market to determine their portfolio's value for periodic reporting and performance measurement purposes. If the market is illiquid, the prevailing market price may not represent the true value of the security and can negatively impact returns/performance.

2. Marking to market is also necessary with repurchase agreements to ensure that the collateral value is adequate to support the funds being borrowed. A lower valuation can lead to a higher cost of funds and decreasing portfolio returns.

Professor's Note: CFA Institute seems to use low liquidity and high liquidity risk interchangeably. I believe you can treat these (liquidity and liquidity risk) as the same concept on the exam, although you should remember that low liquidity means high liquidity risk.

LOS 63.k: Describe the exchange rate risk an investor faces when a bond makes payments in a foreign currency.

If a U.S. investor purchases a bond that makes payments in a foreign currency, dollar returns on the investment will depend on the exchange rate between the dollar and the foreign currency. A depreciation (decrease in value) of the foreign currency will reduce the returns to a dollar-based investor. **Exchange rate risk** is the risk that the actual cash flows from the investment may be worth less than was expected when the bond was purchased.

LOS 63.l: Describe inflation risk and explain why it exists.

Inflation risk refers to the possibility that prices of goods and services in general will increase more than expected. Since fixed-coupon bonds pay a constant periodic stream of interest income, an increasing price level decreases the amount of real goods and services that bond payments will purchase. For this reason, inflation risk is sometimes referred to as purchasing power risk.

LOS 63.m: Explain how yield volatility affects the price of a bond with an embedded option and how changes in volatility affect the value of a callable bond and a putable bond.

Without any volatility in interest rates, a call provision and a put provision have little if any value, assuming no changes in credit quality that impact market values. In general, an increase in the yield/price volatility of a bond increases the values of both put options and call options.

We already saw that the value of a callable bond is less than the value of an otherwise-identical option-free (straight) bond by the value of the call option because the call option is retained by the issuer, not owned by the bondholder. The relation is:

value of a callable bond = value of an option-free bond – value of the call

An increase in yield volatility increases the value of the call option and decreases the market value of a callable bond.

A put option is owned by the bondholder, and the price relation can be described as:

value of a putable bond = value of an option-free bond + value of the put

An increase in yield volatility increases the value of the put option and increases the value of a putable bond.

Therefore, we conclude that increases in interest rate volatility affect the prices of callable bonds and putable bonds in opposite ways. **Volatility risk** for callable bonds is the risk that volatility will increase, and **volatility risk** for putable bonds is the risk that volatility will decrease. *(See Exam Flashbacks #4, #5, and #6.)*

LOS 63.n: Describe the various forms of event risk (e.g., natural catastrophe, corporate takeover/restructuring and regulatory risk) and the components of sovereign risk.

Event risk occurs when something significant happens to a company (or segment of the market) that has a sudden and substantial impact on its financial condition and on the underlying value of an investment. Event risk, with respect to bonds, can take many forms:

- *Disasters* (e.g., hurricanes, earthquakes, or industrial accidents) impair the ability of a corporation to meet its debt obligations if the disaster impacts cash flow adversely. For example, an insurance company's ability to make debt payments may be affected by property/casualty insurance payments in the event of a disaster.
- *Corporate restructurings* (e.g., spin-offs, leveraged buy-outs (LBOs), and mergers) may have an impact on the value of a company's debt obligations by affecting the firm's cash flows and/or the underlying assets that serve as collateral. This may result in bond-rating downgrades and may also affect similar companies in the same industry.
- *Regulatory issues,* such as changes in clean air requirements, may cause companies to incur large cash expenditures to meet new regulations. This may reduce the cash available to bondholders and result in a ratings downgrade. A change in the regulations for some financial institutions prohibiting them from holding certain types of security, such as junk bonds, can lead to a volume of sales that decreases prices for the whole sector of the market.

Sovereign risk refers to changes in governmental attitudes and policies toward the repayment and servicing of debt. Governments may impose restrictions on the outflows of foreign exchange to service debt even by private borrowers. Foreign municipalities may adopt different payment policies due to varying political priorities. A change in government may lead to a refusal to repay debt incurred by a prior regime. Remember, the quality of a debt obligation depends not only on the borrower's ability to repay but also on the borrower's desire or willingness to repay. This is true of sovereign debt as well, and we can think of sovereign risk as having **two components**: a change in a government's willingness to repay and a change in a country's ability to repay. The second component has been the important one in most defaults and downgrades of sovereign debt.

KEY CONCEPTS

1. There are many types of risk associated with fixed income securities:
   - Interest rate risk is defined as the sensitivity of bond prices to changes in interest rates.
   - Call risk is the risk that a bond will be called (redeemed) prior to maturity under the terms of the call provision and that the funds must then be reinvested at the current (lower) yield.
   - Prepayment risk is the risk that the principal on amortizing securities will be repaid early and then must be reinvested at a lower (current) market yield.
   - Yield curve risk is the risk that changes in the shape of the yield curve will negatively impact bond values.
   - Credit risk includes both the risk of default and the risk of decreases in bond value due to a downgrade (reduction in the bond's credit rating).
   - Liquidity risk is the risk that an immediate sale will result in a price below fair value (the prevailing market price).
   - Exchange rate risk is the risk that the foreign exchange value of the currency that a foreign bond is denominated in will fall relative to the home currency of the investor.
   - Volatility risk is the risk that changes in interest rate volatility will affect the value of bonds with embedded options. More volatility decreases callable bond values and increases putable bond values.
   - Inflation risk is the risk that inflation will be higher than expected, eroding the purchasing power of the cash flows from a fixed income security.
   - Event risk is the risk of decreases in a security's value from disasters, corporate restructurings, or regulatory changes that negatively impact the firm.
   - Sovereign risk is the risk that governments may repudiate debt, prohibit debt repayment by private borrowers, or impose general restrictions on currency flows.
2. When a bond's yield is above (below) its coupon rate, it will trade at a discount (premium) to its par value.
3. The interest rate risk of a bond is positively related to its maturity, negatively related to the coupon rate, and is less for bonds with an embedded option (either puts or calls).
4. The price of a callable bond equals the price of an identical option-free bond minus the value of embedded call.
5. The higher the market yield, the lower the interest rate risk.
6. Floating rate bonds have interest rate risk between reset dates and may also differ from par value due to changes in liquidity or in credit risk after they have been issued.
7. The duration of a bond is the approximate percentage price change for a 1 percent change in yield.
8. The percentage price change in a bond = – duration × yield change in percent.
9. When yield curve shifts are not parallel, the duration of a bond portfolio does not capture the true price effects because yields on the various bonds in the portfolio may change by different amounts.
10. A security has more reinvestment risk when it has a higher coupon, is callable, is an amortizing security, or has a prepayment option.
11. A prepayable amortizing security has greater reinvestment risk because of the probability of accelerated principal payments when interest rates (including reinvestment rates) fall.
12. Credit risk includes default risk (the probability of default), downgrade risk (the probability of a reduction in the bond rating), and credit spread risk (uncertainty about the bond's yield spread to Treasuries based on its bond rating).
13. Lack of liquidity can negatively impact periodic portfolio valuation and performance measures for a portfolio and thus can affect a manager even though sale of the bonds is not anticipated.
14. An investor who buys a bond with cash flows denominated in a foreign currency will see the value of the bond decrease if the exchange value of the foreign currency declines (the currency depreciates).
15. If inflation increases unexpectedly, the purchasing power of the cash flows is decreased and bond values fall.
16. Increases in yield volatility increase the value of put and call options embedded in bonds, decreasing the value of a callable bond (because the bondholder is short the call) and increasing the value of putable bonds.

17. Event risk encompasses events that can negatively affect the value of a security, including disasters that negatively impact earnings or diminish asset values, takeovers or restructurings that can negatively impact bondholder claims, and changes in regulation that can negatively affect earnings.
18. Sovereign risk is the risk that actions by a government will restrict outflows to service debt or that a new government may not honor prior debt claims.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #105 from '01–'03 sample exams.

The interest rate risk of a noncallable bond is *most likely* to be positively related to the:
A. risk free rate.
B. bond's coupon rate.
C. bond's time to maturity.
D. bond's yield to maturity.

Exam Flashback # 2

Source: Question #79 from '92 and '96 actual exams and '97 sample exam.

The price volatility of a variable rate note may be reduced by:
A. eliminating any put features.
B. resetting the coupon rate more frequently.
C. reducing the size of the issue.
D. downgrading the quality rating.

Exam Flashback # 3

Source: Question #106 from '98 sample exam.

Which of the following two sources of bond risk have offsetting effects?
A. Default risk and interest rate risk.
B. Reinvestment risk and default risk.
C. Interest rate risk and reinvestment risk.
D. Exchange rate risk and volatility risk.

Exam Flashback # 4

Source: Question #107 from '99 sample exam.

Which of the following is **NOT** a component of call risk for a bond investor?
A. The cash flow pattern of a callable bond is not known with certainty.
B. When the issuer calls a bond, the investor is exposed to reinvestment risk.
C. The value of a callable bond drops when expected interest rate volatility decreases.
D. The capital appreciation potential of a callable bond is lower than a noncallable bond.

Exam Flashback # 5

Source: Question #102 from '99 sample exam.

Which of the following risks for a bond is most directly determined by whether that bond has an embedded option?
A. Credit risk.
B. Market risk.
C. Volatility risk.
D. Interest rate risk.

Exam Flashback # 6
Source: Question #16 from '90 actual exam (revised).

A company issues the following bonds on June 1, 1990:

	Series A	Series B
Par value	$100 million	$100 million
Rating	AA	AA
Maturity	June 1, 2010	June 1, 2010
Call date	June 1, 1998	Non-callable
Call price	100	---

If both bonds have the same market liquidity, the yield-to-maturity and volatility risk, respectively, on the Series A bond compared to the Series B bond would be:
A. lower, lower.
B. lower, higher.
C. higher, lower.
D. higher, higher.

Concept Checkers: Risks Associated With Investing in Bonds

1. A bond with a 7.3% yield has a duration of 5.4 and is trading at $985.00. If the yield decreases to 7.1%, the new bond price is *closest to*:
   A. $974.40.
   B. $1,038.30.
   C. $995.60.
   D. $1091.40.

2. If interest rate volatility *increases*, which of the following bonds will experience a price *decrease*?
   A. A callable bond.
   B. A putable bond.
   C. A zero-coupon, option-free bond.
   D. An option-free, 4% coupon bond.

3. A noncallable, AA-rated, 5-year zero-coupon bond with a yield of 6 percent has all of the following **EXCEPT:**
   A. interest rate risk.
   B. inflation risk.
   C. reinvestment risk.
   D. default risk.

4. The current price of a bond is 102.50. If interest rates change by 0.5 percent, the value of the bond price changes by 2.50. What is the duration of the bond?
   A. 5.00.
   B. 2.44.
   C. 4.88.
   D. 2.50.

5. Which of the following bonds has the greatest interest rate risk?
   A. A 5% 10-year callable bond yielding 4%.
   B. A 5% 10-year putable bond yielding 6%.
   C. A 5% 10-year option-free bond yielding 4%.
   D. A 5% 10-year option-free bond yielding 6%.

6. A floating-rate security will have the greatest duration:
   A. the day before the reset date.
   B. the day after the reset date.
   C. just prior to maturity because that is the largest cash flow.
   D. never—floating-rate securities have a duration of zero.

7. The duration of a bond is 5.47, and its current price is \$986.30. Which of the following is the best estimate of the bond price change if interest rates *increase* by 2 percent?
   A. –\$109.40.
   B. –\$107.90.
   C. \$107.90.
   D. \$109.40.

8. A straight 5 percent bond has two years remaining to maturity and is priced at \$981.67. A *callable bond* that is the same in every respect as the straight bond, except for the call feature, is priced at \$917.60. With the yield curve flat at 6 percent, what is the value of the embedded call option?
   A. –\$82.40.
   B. \$45.80.
   C. \$64.07.
   D. \$101.00.

9. A straight 5 percent coupon bond has two years remaining to maturity and is priced at \$981.67 (\$1,000 par value). A putable bond that is the same in every respect as the straight bond except that the put provision is priced at 101.76 (percent of par value). With the yield curve flat at 6 percent, what is the value of the embedded put option?
   A. –\$35.93.
   B. –\$17.60.
   C. \$17.60.
   D. \$35.93.

10. All of the following are possible examples of event risk with respect to fixed-income securities, **EXCEPT**:
    A. an earthquake.
    B. a change in rate regulation.
    C. a Federal Reserve decrease in money supply.
    D. one firm's acquisition by another.

11. Which of the following 5-year bonds has the *highest* interest rate risk?
    A. A floating-rate bond.
    B. A zero-coupon bond.
    C. A callable 5% fixed-coupon bond.
    D. An option-free 5% fixed-coupon bond.

12. An investor is concerned about interest rate risk. Which of the following four bonds (similar except for yield and maturity) has the *least* interest rate risk? The bond with:
    A. 5% yield and 10-year maturity.
    B. 5% yield and 20-year maturity.
    C. 6% yield and 10-year maturity.
    D. 6% yield and 20-year maturity.

13. Which of the following statements about the risks of bond investing is **TRUE**?
    A. A bond rated AAA has no credit risk.
    B. A bond with call protection has volatility risk.
    C. A U.S. Treasury bond has no exchange rate risk.
    D. A U.S. Treasury bond has no reinvestment risk.

14. Which of the following securities will have the *least* reinvestment risk for a long-term investor?
    A. A 10-year, zero-coupon bond.
    B. A 6-month Treasury bill.
    C. A 30-year, prepayable amortizing bond.
    D. A 10-year, 4% debenture.

15. Which of the following does a 2-year, zero-coupon U.S. Treasury note **NOT** have?
    A. Inflation risk.
    B. Interest rate risk.
    C. Currency risk.
    D. Volatility risk.

ANSWERS – EXAM FLASHBACKS

1. **C** As the time to maturity increases, more of the bond's cash flows occur farther out in the future. As a general rule, a given change in the discount rate will have a larger impact on cash flows that are farther out versus nearer in. Hence, interest rate risk is positively related to maturity.

2. **B** The more often you reset a variable rate bond's coupon, the closer it will track its par value. Since a bond's price will change as the market interest rate moves away from the coupon rate, by resetting the coupons frequently you reduce price volatility.

3. **C** As interest rates rise, the price of the bond will fall (interest rate risk). However, as interest rates rise, the amount received from coupon reinvestment will rise, offsetting the effects of interest rate risk.

4. **C** Callable bond price = non-callable price – call option price. If volatility drops, the option value will fall, causing the callable bond price to rise, not fall as C says.

5. **C** In general, option prices are affected by the volatility of the underlying instrument. As interest rate volatility rises, the value of the embedded option will rise.

6. **D** Bond A is callable and must pay higher yields to compensate investors for the call risk. In other words, all else the same, the price of Bond A must be below that of Bond B (yield is higher). In addition, Bond B would have no volatility risk because it is not callable and does not contain an embedded option. Please be careful about the definition of volatility risk—volatility risk does not refer to interest rate risk per se, but refers to the impact that a change in the volatility of interest rates will have on the value of an embedded option. We have altered this question from its original form to include the subquestion on volatility and make it more difficult—in keeping with more recent CFA exams.

ANSWERS – CONCEPT CHECKERS: RISKS ASSOCIATED WITH INVESTING IN BONDS

1. **C** The percentage price change, based on duration is equal to –5.4 × (–0.2%) = 1.08%. The new price is 1.0108 × 985 = $995.64.

2. **A** An increase in volatility will increase the value of the call option and decrease the value of a callable bond. A putable bond will increase in value. The value of option-free bonds will be unaffected.

3. **C** A zero-coupon bond, as a security, has no reinvestment risk because there are no cash flows prior to maturity that must be reinvested. A double-A bond has some (small) default risk. Zero-coupon bonds have the most interest rate risk for a given maturity. All bonds that are not indexed to inflation rates have inflation risk, or purchasing power risk.

4. **C** The duration is computed as follows:

$$\text{duration} = \frac{\text{percentage change in price}}{\text{change in yield as a decimal}} = \frac{2.50/102.5}{0.005} = \frac{2.44\%}{0.5\%} = 4.88$$

5. **C** Embedded options reduce duration/interest rate risk. The straight bond with the lower coupon will have greater duration than the straight bond with the higher coupon.

6. **B** The duration of a floating-rate bond is higher the greater the time lag until the next coupon payment/reset date. The greatest duration/interest rate risk is, therefore, immediately after the coupon has been reset.

7. **B** The approximate dollar change in price is computed as follows:

   dollar price change = –5.47 × 0.02 × 986.30 = –$107.90

8. **C** The option value is the difference between the value of an option-free bond and the corresponding price of the callable bond. Its value is computed as:

   call option value = \$981.67 – \$917.60 = \$64.07

9. **D** The value of the embedded put option is the difference between the price of the putable bond and the price of the straight bond. So it is computed as:

   option value = \$1,017.60 – \$981.67 = \$35.93

10. **C** Event risk refers to events that can impact a firm's ability to pay its debt obligations that are separate from market risks. The Fed's actions can impact interest rates, but this is a market risk factor, not event risk.

11. **B** The zero-coupon bond will have the greatest duration of any of the four bonds and, as such, will be subject to the greatest interest rate risk.

12. **C** Interest rate risk is *inversely* related to the yield and directly related to maturity. All else equal, the lower the yield, the greater the interest rate risk. All else equal, the longer the maturity, the greater the interest rate risk. This bond has the higher yield and the shorter maturity, and thus has the lowest interest rate risk.

13. **B** A Treasury bond pays semiannual coupon interest and, therefore, has reinvestment risk. A triple-A rated bond can lose its AAA rating, so it has downgrade risk, a component of credit risk. Any bond can have exchange rate risk if the security holder's returns are measured in a different currency. A bond with a call feature has volatility risk even when the call cannot be exercised immediately. The call feature still has value (to the issuer), and its value will be affected by volatility changes.

14. **A** A 10-year, zero-coupon bond has no cash flows prior to maturity to reinvest while the entire amount invested in 6-month bills must be reinvested twice each year.

15. **D** It will have both inflation (purchasing power) risk and interest rate risk. It will have currency risk to non-U.S. dollar investors. Volatility risk only applies to bonds with embedded options.

The following is a review of the Debt Investments: Basic Concepts principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

Overview of Bond Sectors and Instruments

Study Session 14

Exam Focus

This review introduces the various types of fixed income securities and a fair amount of terminology relating to fixed income securities. Pay special attention to the mechanics of these securities; that is, how they pay, when they pay, and what they pay. The additional information is nice, but likely not crucial. Try to gain enough understanding of the terms listed in the learning outcome statements so that you will understand them when they are used in a question. Much of this material is unlikely to be tested by itself, however, knowing the basics about Treasury securities, mortgage-backed securities, and municipal securities is important as a foundation for much of the material on debt securities that follows, as well as for the more detailed material on fixed income valuation and risk that is contained in the Level 2 and Level 3 curriculum.

LOS 64.a: Describe the different types of international bonds (e.g., Eurobonds, global bonds, sovereign debt).

We can separate a country's bond markets into domestic and foreign bond markets. The domestic market refers to bonds issued by companies in that country and traded in that country. The foreign bond market refers to bonds issued by companies based outside the country where the bonds are issued and traded.

Foreign bonds can be denominated in any currency and may be privately placed or publicly issued. They may be issued by private corporations, national governments and their agencies, and supranationals. Supranationals are agencies such as the World Bank, which are created by an international group of governments generally as a catalyst for economic development.

Eurobonds are issued outside the legal jurisdiction of any one country. Thus, they are not usually registered through a regulatory agency. They are offered simultaneously to investors in several countries by a group (syndicate) of international investment and merchant bankers. Eurobonds are identified by the currency in which they are denominated (e.g., Eurodollar bonds, Euroyen bonds, etc.).

The term **global bond** is used to refer to a bond that trades in the Eurobond market and is also traded in the foreign bond market of at least one country.

Sovereign debt is issued by central governments. U.S. Treasury bonds are sovereign debt. Governments can issue domestic, foreign, or Eurobonds to raise funds. In addition, they can also borrow directly from banks to finance their expenditures. (*See Exam Flashbacks #1 and #2.*)

LOS 64.b: Describe the types of securities issued by the U.S. Department of the Treasury (e.g. bills, notes, bonds, and inflation protection securities), differentiate between on-the-run and off-the-run Treasury securities, discuss how stripped Treasury securities are created, and distinguish between coupon strips and principal strips.

Treasury securities (Treasuries) are issued by the U.S. Treasury. Because they are backed by the full faith and credit of the U.S. government, they are considered to be free from credit risk (though they're still subject to

interest rate/price risk). The Treasury issues three distinct types of securities: bills, notes and bonds, and inflation-protected securities.

Treasury bills (T-bills) have maturities of less than one year and do not make explicit interest payments, paying only the face (par) value at the maturity date. T-bills are sold at a discount to par value and interest is received when the par value is paid at maturity (like zero-coupon bonds). The interest on T-bills is sometimes called *implicit interest* since the interest (difference between the purchase price and the par value) is not made in a separate "explicit" payment, as it is on bonds and notes. Securities of this type are known as *pure discount* securities.

- There are *three maturity cycles*: 28, 91, and 182 days, adjustable by one day (up or down) due to holidays. They are also known as 4-week, 3-month, and 6-month T-bills, respectively.

- Periodically, the Treasury also issues *cash management* bills with maturities ranging from a few days to six months to help overcome temporary cash shortages prior to the quarterly receipt of tax payments.

Treasury notes and **Treasury bonds** pay semiannual coupon interest at a rate that is fixed at issuance. Notes have original maturities of 2, 3, 5, and 10 years. Bonds have original maturities of 20 or 30 years. Although many bonds are still outstanding and still traded, the Treasury is not currently issuing new bonds.

Prior to 1984, some Treasury bonds were issued that are callable at par five years prior to maturity. The Treasury has not issued callable bonds since 1984.

Treasury bond and note prices in the secondary market are quoted in percent and 32nds of one percent of face value. A quote of 102-5 (sometimes 102:5) is 102 percent plus five 32nds percent of par, which for a $100,000 face value T-bond, translates to a price of:

$$\left[102+\frac{5}{32}\right]\%\times\$100{,}000=1.0215625\times\$100{,}000=\$102{,}156.25$$

Since 1997, the Treasury has issued **Treasury Inflation-Protected Securities** (TIPS). Currently, only notes are offered but some inflation-protected 20- and 30-year bonds were previously issued and trade in the secondary market. The details of how TIPS work are:

- TIPS make semiannual coupon interest payments at a rate fixed at issuance, just like notes and bonds.
- The par value of TIPS begins at $1,000 and is adjusted semiannually for changes in the Consumer Price Index (CPI). Even if the there is deflation (falling price levels), the par value can never be adjusted to below $1,000. The fixed coupon rate is paid semiannually as a percentage of the *inflation adjusted par value.*
- Any increase in the par value from the inflation adjustment is taxed as income in the year of the adjustment:

$$\text{TIPS coupon payment} = \text{inflation-adjusted par value} \times \frac{\text{stated coupon rate}}{2}$$

For example, consider a $100,000 par value TIPS with a 3 percent coupon rate, set at issuance. Six months later the *annual* rate of inflation (CPI) is 4 percent. The par value will be increased by one-half of the 4 percent (i.e., 2 percent) and will be 1.02 × 100,000 = $102,000.

The first *semiannual* coupon will be one-half of the 3 percent coupon rate times the inflation adjusted par value: 1.5% × 102,000 = $1,530. Any percentage change in the CPI over the next 6-month period will be used to adjust the par value from $102,000 to a new inflation-adjusted value, which will be multiplied by 1.5 percent to compute the next coupon payment. *(See Exam Flashback #3.)*

On-the-Run and Off-the-Run Treasury Securities

Treasury issues are divided into two categories based on their vintage:

- **On-the-run issues** are the most recently auctioned Treasury issues.
- **Off-the-run issues** are older issues that have been replaced (as the most traded issue) by a more recently auctioned issue. Issues replaced by several more recent issues are known as *well off-the-run* issues.

The distinction is that the on-the-run issues are more actively traded and therefore more liquid than off-the-run issues.

Stripped Treasury Securities

Since *the U.S. Treasury does not issue zero-coupon notes and bonds*, investment bankers began stripping the coupons from Treasuries to create zero-coupon securities of various maturities to meet investor demand. These securities are termed **stripped Treasuries or Treasury strips.** In 1985, the Treasury introduced the Separate Trading of Registered Interest and Principal Securities (STRIPS) program. Under this program, the Treasury issues coupon-bearing notes and bonds as it normally does, but then it allows certain government securities dealers to buy large amounts of these issues, strip the coupons from the principal, repackage the cash flows, and sell them separately as zero-coupon bonds, at discounts to par value.

For example, a 10-year T-note has 20 coupons and one principal payment; these 21 cash flows can be repackaged and sold as 21 different zero-coupon securities. The stripped securities (*Treasury strips*) are divided into two groups:

- **Coupon strips** (denoted as *ci*) refers to strips created from coupon payments stripped from the original security.
- **Principal strips** refers to bond and note principal payments with the coupons stripped off. Those derived from stripped bonds are denoted *bp* and those from stripped notes *np*.

Professor's Note: While the payments on coupon strips and principal strips with the same maturity date are identical, certain countries treat them differently for tax purposes, and they often trade at slightly different prices.

STRIPS are taxed by the IRS on their implicit interest (movement toward par value), which, for fully taxable investors, results in negative cash flows in years prior to maturity. The Treasury STRIPS program also created a procedure for *reconstituting* Treasury notes and bonds from the individual pieces.

LOS 64.c: Describe a mortgage-backed security, and explain the cash flows for a mortgage-backed security, define prepayments, and explain prepayment risk.

Mortgage-backed securities (MBSs) are backed (secured) by pools of mortgage loans, which not only provide *collateral* but also the *cash flows* to service the debt. A mortgage-backed security is any security where the collateral for the issued security is a pool of mortgages.

The cash flows from a mortgage or from a pool of mortgages are different from the cash flows of a coupon bond. Mortgage loans are amortizing loans in that they make a series of equal payments consisting of the periodic interest on the outstanding principal at the beginning of the period and a partial repayment of the principal amount. Residential real estate mortgages are typically for 30 years and consist of 360 equal monthly payments. In the early years, interest is the largest portion of the payment and the final payment after 30 years is almost all principal.

Professor's Note: Amortizing loans and amortization schedules are covered in Study Session 2.

Prepayments and Prepayment Risk

The cash flows of a mortgage passthrough security mirror those of a mortgage loan, with some interest and some principal. Any payment of principal in excess of the scheduled principal repayment (the principal component of the regular level mortgage payment) is called a **prepayment.** A prepayment on a mortgage loan that is less than the entire remaining principal amount is called a *curtailment.*

Most mortgage loans allow the borrower to prepay principal at any time and in any amount without a penalty. Therefore, mortgage passthrough securities are subject to the **prepayment risk** described in the previous topic review. Since homeowners are more likely to prepay principal when interest rates fall (are low), the prepayment option is similar to a call feature. A prepayment option subjects the security holder to more reinvestment risk than would be the case with a mortgage that could not be prepaid (or only prepaid with a significant penalty).

In summary, the monthly cash flows into a mortgage pool underlying a passthrough security consist of three components:

- Monthly interest.
- Scheduled monthly principal payments.
- Prepayments, or principal repayments in excess of scheduled principal payments.

LOS 64.d: Describe the types and characteristics of securities issued by federal agencies (including mortgage passthroughs and collateralized mortgage obligations).

Agency bonds are debt securities issued by various agencies and organizations of the U.S. Government, such as the Federal Home Loan Bank (FHLB). Most agency issues are *not* obligations of the U.S. Treasury and technically should not be considered the same as Treasury securities. Even so, they are very high quality securities that have almost no risk of default. There are two types of federal agencies:

- *Federally related institutions,* such as the Government National Mortgage Association (Ginnie Mae) and the Tennessee Valley Authority (TVA), which are owned by the U.S. Government and are exempt from Securities Exchange Commission (SEC) registration. In general, these securities are backed by the full faith and credit of the U.S. Government, except in the case of the TVA and Private Export Funding Corporation. Essentially, these securities are free from credit risk.
- *Government sponsored enterprises* (GSEs) include the Federal Farm Credit System, the Federal Home Loan Bank System, the Federal National Mortgage Association (Fannie Mae), the Federal Home Loan Bank Corporation (Freddie Mac), and the Student Loan Marketing Association (Sallie Mae). These are privately owned, but publicly chartered organizations, and were created by the U.S. Congress. They issue their securities directly in the marketplace and expose investors to some (albeit very little) credit risk.

The following are types of securities issued by federal agencies.

Debentures are securities that are not backed by collateral (i.e., they are unsecured). GSEs commonly issue debentures. These are of many maturity structures and can be coupon interest paying securities or discount securities (referred to as bills).

A **mortgage passthrough security** is created by pooling a number of mortgages together, usually several thousand mortgages.

- Shares of such mortgage pools are sold in the form of *participation certificates* representing ownership of a fractional share of the pool of underlying mortgages. A $100,000 (par value) participation certificate would represent a 1% interest in the cash flows from a $10 million (face value) pool of mortgages.
- The interest and principal payments made by the homeowners whose mortgages are in the pool are collected and *passed through* to investors after deducting a small amount for administrative and servicing fees.

Ginnie Mae, Freddie Mac, and Fannie Mae all issue passthrough securities and guarantee timely payment of interest and principal.

Collateralized mortgage obligations (CMOs) are created from mortgage passthrough certificates and referred to as derivative mortgage-backed securities, since they are derived from a simpler MBS structure. CMOs have a more complex structure than mortgage passthroughs. As we will explain shortly, a CMO issue has different "tranches," each of which has a different type of claim on the cash flows from the pool of mortgages (i.e., their claims are not just a fractional claim on the total cash flows from the pool.

Professor's Note: Tranche is from the French word for "slice." In finance, when a security issue consists of different classes of securities with differing claims and especially with differing risks, the different classes of securities are called tranches. You will likely run into this term only in reference to the different classes of securities that make up a CMO.

LOS 64.e: State the motivation for creating a collateralized mortgage obligation, describe the types of securities issued by municipalities in the United States, and distinguish between tax-backed debt and revenue bonds.

The **motivation for creating collateralized mortgage obligations** (CMOs) is to *redistribute the prepayment* risk inherent in mortgage passthrough securities and/or *create securities with various maturity ranges*. The CMO structure takes the cash flows from the mortgage pool and, in a simple structure, allocates any principal payments (both scheduled payments and prepayments) sequentially over time to holders of different CMO tranches, rather than equally to all security holders. Creating a CMO does not alter the *overall* risk of prepayment, it redistributes prepayment risk.

An example of a simple *sequential* CMO structure with three tranches will help to illustrate the process. Assume that three tranches are created out of a passthrough security. Let's call them Tranches I, II, and III. They receive interest on the basis of their outstanding par values. The following are the details of the payments to each of the three tranches.

- Tranche I (the *short*-term segment of the issue) receives net interest on outstanding principal and all of the principal payments from the mortgage pool until it is completely paid off.
- Tranche II (the *intermediate*-term) receives its share of net interest and starts receiving all of the principal payments after Tranche I has been completely paid off. Prior to that, it only receives interest payments.
- Tranche III (the *long*-term) receives monthly net interest and starts receiving all principal repayments after Tranches I and II have been completely paid off. Prior to that it only receives interest payments.

Tranche I has the shortest expected maturity and may appeal to an investor with a preference for securities with a shorter time horizon, who previously could not participate in the mortgage-backed securities market. Other structures, with prepayments primarily affecting only some of the tranches, are used to redistribute prepayment risk. The tranches with less prepayment risk will become more attractive to some investors. Investors better able to bear prepayment risk will find the tranches with higher prepayment risk attractive.

As a general rule, CMOs are created to satisfy a broader range of investor risk/return preferences—making investing in mortgage-backed securities more appealing to a wider audience and decreasing overall borrowing costs.

Types of Securities Issued By Municipalities

Debt securities issued by state and local governments in the U.S. are known as *municipal bonds* (or "munis" for short). *Municipal bonds* are issued by states, counties, cities, and other political subdivisions (such as school, water, or sewer districts). These bonds are often issued as *serial bonds*, that is, a larger issue is divided into a series of smaller issues, each with its own maturity date and coupon rate.

Municipal bonds are often referred to as *tax-exempt* or *tax-free* bonds, since the coupon interest is exempt from federal income taxes. Note that, while interest income may be tax free, realized capital gains are not. They are subject to normal capital gains taxes at the federal level. However, not all municipal bonds are tax exempt; some are taxable:

- *Tax exempt.* Different states tax municipal securities differently; the vast majority of states treat *their own bonds* (i.e., those issued within the state) as tax exempt, but consider the interest income earned on out-of-state bonds as fully taxable. Thus, the interest income earned on most in-state bonds held by a resident of that state is free from *both* state and federal income tax. Such bonds are referred to as *double tax free.*
- *Taxable.* A municipal bond must meet certain federal standards in order to qualify for the tax-exempt status. If they don't, the bonds are considered "taxable" and the *interest income on these bonds is subject to federal income tax* (they could still be exempt from state taxes). *Taxable municipal bonds are the exception* rather than the rule, as most municipal issues are exempt from federal taxes.

An opinion as to the tax-exempt status of the bonds, typically by a well-respected law firm specializing in municipal bond issues, is provided to purchasers when the bonds are issued.

Tax-Backed Debt and Revenue Bonds

Tax-backed bonds, also called general obligation (GO) bonds, are backed by the full faith, credit, and *taxing power* of the issuer. Tax-backed debt is issued by school districts, towns, cities, counties, states, and special districts, and include the following types:

- *Limited tax GO debt* is subject to a statutory limit on taxes that may be raised to pay off the obligation.
- *Unlimited tax GO debt*, the most common type of GO bond, is secured by the full faith and credit of the borrower and backed by its unlimited taxing authority, which includes the ability to impose individual income tax, sales tax, property tax, and corporate tax. This is the more secure form of GO.
- *Double-barreled* bonds, a special class of GOs, are backed not only by the issuing authority's taxing power, but also by additional resources that could include fees, grants, and special charges that fall outside the general fund.
- *Appropriation-backed obligations* are also known as *moral obligation bonds.* States sometimes act as a back up source of funds for issuers during times of shortfall. However, the state's obligation is not legally binding, but is a "moral obligation." The state may appropriate funds from its general fund. This *moral pledge* enhances the security of such bonds.
- Debt supported by *public credit enhancement* programs possess a guarantee by the state or federal government, which is a legally enforceable contract and is used normally to assist the state's school system.

Revenue bonds are supported only through revenues generated by projects that are funded with the help of the original bond issue. For example, revenue bonds can be issued to fund transportation systems, housing projects, higher education, health care, sports arenas, harbors, and ports. These bonds fall outside GO debt limits and do not require voter approval.

The distinction between a general obligation and a revenue bond is important for a bondholder, because the issuer of a revenue bond is obligated to pay principal and interest *only if a sufficient level of revenue is generated* by the project. If the funds aren't there, the issuer does not make payments on the bond. In contrast, general obligation bonds are required to be serviced in a timely fashion irrespective of the level of tax income generated

by the municipality. At issuance, revenue bonds typically involve more risk than general obligation bonds and, therefore, provide higher yields. (*See Exam Flashback #4.*)

LOS 64.f: Describe insured bonds and prerefunded bonds.

Insured bonds carry the guarantee of a third party that all principal and interest payments will be made in a timely manner. The third-party guarantee (insurance) typically cannot be canceled; it is good for the life of the bond. There are several firms that specialize in providing insurance for municipal bond issues. Municipal bond insurance results in higher ratings, usually AAA, which reduces the required yield and improves the liquidity of the bonds. Insured bonds are especially common in the revenue bond market but the general obligation bonds of smaller municipal issuers are often insured to broaden their appeal to investors.

Prerefunded bonds are bonds for which Treasury securities have been purchased and placed in a special escrow account in an amount sufficient to make all the remaining required bond payments. The Treasury securities' income and principal payments must be sufficient to fund the municipal bond's required payments until maturity or through the first call date. Bonds that are prerefunded have little or no credit risk and are likely to receive a rating of AAA.

LOS 64.g: Summarize the bankruptcy process and bondholder rights, explain the factors considered by rating agencies in assigning a credit rating to a corporate debt instrument, and describe secured debt, unsecured debt, and credit enhancements for corporate bonds.

Bonds issued by corporations have priority over both common and preferred stock in receiving cash distributions from the firm (bond interest paid before dividends) and priority in receiving distributions in the event of firm insolvency leading to bankruptcy.

The Bankruptcy Reform Act of 1978 is the primary bankruptcy law in the U.S. Holders of debt instruments have a priority over stockholders, and there may be several priority classes among the creditors themselves, depending on the seniority of their claims. Bankruptcy laws encompass two different types of bankruptcies:

- *Liquidation.* When a corporate entity cannot function as a going concern, it is liquidated and all its assets are distributed to the claimants of the firm, including bondholders and shareholders (if funds are sufficient). The corporate entity does not remain in existence. This is Chapter 7 bankruptcy. As much as possible, the creditors receive the assets of the company on the basis of *absolute priority rule.* This means that senior creditors get paid before junior creditors (subordinated debt) who get paid before the holders of common and preferred stock.
- *Reorganization.* In this case, a corporation survives but with a capital structure that can be very different from its original structure. Some claimants receive securities in the reorganized corporation, and some receive cash; some receive both. However, most creditors lose some value. This is Chapter 11 bankruptcy. The *absolute priority rule* is not strictly adhered to in reorganizations.
- An important function of the bankruptcy act is to provide a corporation protection from its creditors while it decides which is the most advantageous choice: liquidation or reorganization. There are two types of bankruptcy filings.
    - A *bankruptcy petition*, if filed in the court by the corporation, results in a *voluntary bankruptcy.* In this case, the corporation becomes a *debtor in possession*, and operates under the court's supervision.
    - A bankruptcy petition, if filed by a corporation's creditors, results in an *involuntary bankruptcy.*

Rating Agencies and Credit Ratings

Rating agencies, such as Moody's and S&P, rate specific debt issues of corporations. Some of the factors they consider are quantitative, but many are qualitative. Even quantitative factors can be somewhat subjective. The ratings are issued to indicate the relative probability that all promised payments on the debt will be made over the

life of the security and, therefore, must be forward looking. Ratings on longer-term bonds will consider factors that may come into play over at least one full economic cycle.

Some of the *firm-specific* factors considered are:

- Past repayment history.
- Quality of management, ability to adapt to changing conditions.
- The industry outlook and firm strategy.
- Overall debt level of the firm.
- Operating cash flow, ability to service debt.
- Other sources of liquidity (cash, salable assets).
- Competitive position, regulatory environment, union contracts/history.
- Financial management and controls.
- Susceptibility to event risk and political risk.

Some factors *specific to a particular debt issue* are:

- Priority of the claim being rated.
- Value/quality of any collateral pledged to secure the debt.
- The covenants of the debt issue.
- Any guarantees or obligations for parent company support.

Professor's Note: It may help to remember the primary factors as all Cs: Character of the issuer, Capacity to repay, the Collateral provided, and the Covenants of the debt issue.

Secured Debt, Unsecured Debt, and Credit Enhancements for Corporate Bonds

Secured debt is backed by the pledge of assets/collateral, which can take the following forms:

- *Personal property* (e.g., machinery, vehicles, patents).
- *Financial assets* (e.g., stocks, bonds, notes).
- These assets are marked to market from time to time to monitor their liquidation values. Covenants may require a pledge of more assets if values are insufficient.
- *Real property* (e.g., land and buildings).
- In all of these cases, the bondholder holds a lien on the pledged property. In the case of default, the lien holder can sell the property and use the proceeds to satisfy the obligations of the borrower. In most cases of default, some mutual agreement will be reached for a new structure, but the bondholders' claim on the pledged assets significantly strengthens their position in renegotiation.

Unsecured debt is not backed by any pledge of specific collateral. Unsecured bonds are referred to as *debentures*. They represent a general claim on any assets of the issuer that have not been pledged to secure other debt. If pledged assets generate funds upon liquidation in excess of the obligation, then these excess funds are available for satisfying the claims of unsecured debt holders. *Subordinated debentures* have claims that are satisfied after (subordinate to) the claims of *senior debt*.

Credit enhancements are the guarantees of others that the corporate debt obligation will be paid in a timely manner. Typically, they take one of the following forms:

- *Third-party* guarantees that the debt obligations will be met. Often, parent companies guarantee the loans of their affiliates and subsidiaries.
- *Letters of credit* are issued by banks and guarantee that the bank will advance the funds to service the corporation's debt.
- *Bond insurance* can be obtained from firms that specialize in providing it.

When analyzing credit-enhanced debt, analysts should focus on the financial strength of both the corporation issuing the debt and the financial strength of the party providing credit enhancement. The protection to the bond holder is no better than the promise of the entity offering the credit enhancement. A decrease in the creditworthiness of the guarantor (enhancer) can lead to a rating downgrade of the debt issue.

LOS 64.h: Distinguish between a corporate bond and a medium-term note.

Professors Note: Be careful here, medium-term notes are not medium-term and not necessarily notes!

Corporate bond issues typically: (1) are sold all at once, (2) are sold on a firm-commitment basis whereby an underwriting syndicate guarantees the sale of the whole issue, and (3) consist of bonds with a single coupon rate and maturity.

Medium-term notes (MTNs) differ from a regular corporate bond offering in all of these characteristics.

MTNs are registered under SEC Rule 415 (*shelf registration*) which means that they need not be sold all at once. Once registered, such securities can be "placed on the shelf" and sold in the market over time at the discretion of the issuer. MTNs are sold over time, with each sale satisfying some minimum dollar amount set by the issuer, typically one million dollars and up.

MTNs are issued in various maturities, ranging from 9 months to periods as long as 100 years. The issuer provides *maturity ranges* (e.g., 18 months to 2 years) for MTNs that they wish to sell and provides yield quotes for those ranges, typically as a spread to comparable maturity Treasury issues. Investors interested in purchasing the notes make an offer to the issuer's agent, specifying the face value and an exact maturity within one of the ranges offered. The agent then confirms the issuer's willingness to sell those MTNs and effects the transaction.

The offering is done by the issuer's agent on a *best-efforts* basis. There is no firm commitment on the agent's part to sell a specific amount of bonds.

MTNs can have fixed or floating-rate coupons, can be denominated in any currency, and can have special features, such as calls, caps, floors, and non-interest rate indexed coupons. The notes issued can be combined with derivative instruments to create the special features that an investor requires. The combination of the derivative and notes is called a *structured security.*

LOS 64.i: Describe a structured note, explain the motivation for their issuance by corporations, describe commercial paper, and distinguish between directly-placed paper and dealer-placed paper, and describe the salient features, uses and limitations of bank obligations (negotiable CDs and bankers acceptances).

A structured note is a debt security created when the issuer combines a typical bond or note with a derivative. This is done to create a security that has special appeal to some institutional investors. As with any innovative debt security, the motivation to issue them is to lower overall borrowing costs. The fact that the targeted institutional investors face restrictions on the types of securities they can purchase leads to the reduction in borrowing costs and structured securities allow them to avoid these restrictions and essentially "break the rules."

As an example, consider an institutional investor that is prohibited from owning equity or derivative securities. An issuer could create a structured note where the periodic coupon payments were based on the performance of an equity security or an equity index. This structured note would still be a debt security, but would produce returns closer to holding the equity index itself. The mechanics of creating this security would be to issue a debt security and combine it with an *equity swap*. An equity swap is a derivative that requires the payment of a fixed rate of interest (the coupon rate on the bond here), and pays its owner the rate of return on the equity or equity index each period. By combining the bond with the equity swap, a structured note is created that pays the percentage rate of return on the equity semiannually instead of paying a fixed coupon payment.

We will cover equity swaps, interest rate swaps, and other derivatives commonly used to create structured notes in a subsequent study session. For our purposes here, it is sufficient that you understand that structured notes are created by combining regular debt with derivative securities to make a "debt security" that allows certain institutional investors to get around restrictions they face and thereby reduce the borrowing costs of the company creating the structured note.

Commercial Paper: Directly-Placed and Dealer-Placed Paper

Commercial paper is a short-term unsecured debt instrument used by corporations to borrow money at rates lower than bank rates. Commercial paper is issued with maturities of 270 days or less, since debt securities with maturities of 270 days or less are exempt from SEC registration. It is issued with maturities as short as 2 days, with most issues being in the 2-day to 90-day range.

Like T-bills, commercial paper is typically issued as a pure discount security and makes a single payment equal to the face value at maturity. There is no active secondary market in commercial paper and most buyers hold commercial paper until maturity.

Commercial paper is generally issued by corporations with relatively strong credit and the proceeds are often used to finance credit given to the firm's customers or to finance inventories. Finance subsidiaries of manufacturing firms issue commercial paper to fund customers' purchases of the parent company's products. Issuers often keep unused bank lines of credit in place to use in case new paper cannot be issued to generate the funds needed to pay off maturing paper.

Directly-placed paper is commercial paper that is sold to large investors without going through an agent or broker-dealer. Large issuers will deal with a select group of regular commercial paper buyers who customarily buy very large amounts.

Dealer-placed paper is sold to purchasers through a commercial-paper dealer. Most large investment firms have commercial paper desks to serve their customers' needs for short-term cash-management products.

Negotiable CDs and Bankers Acceptances

Certificates of deposit (CDs) are issued by banks and sold to their customers. They represent a promise by the bank to repay a certain amount plus interest and, in that way, are similar to other bank deposits. In contrast to regular bank deposits, CDs are issued in specific denominations and for specified periods of time that can be of any length. In the U.S., CDs are insured by the Federal Deposit Insurance Corporation (FDIC) for up to $100,000 in the event the issuing bank becomes insolvent. Amounts above $100,000 are not insured and are, therefore, only as secure as the bank that issues the CD.

Typical bank CDs in the U.S. carry a penalty to the CD owner if the funds are withdrawn earlier than the maturity date of the CD. **Negotiable CDs**, however, permit the owner to sell the CD in the secondary market at any time. Negotiable CDs issued in the U.S. by U.S. banks are termed domestic CDs and U.S. dollar denominated CDs issued by foreign banks and branches of U.S. banks outside the U.S. are termed Eurodollar CDs. Negotiable CDs have maturities ranging from days up to 5 years. The interest rate paid on them is called the London Interbank Offering Rate because they are primarily issued by banks' London branches.

Bankers acceptances are essentially guarantees by a bank that a loan will be repaid. They are created as part of commercial transactions, especially international trade. As an example, consider an importer who agrees to pay for goods shipped to him by an exporter, 45 days after the goods are shipped. The importer goes to his bank and gets a letter of credit stating that the bank will guarantee the payment, say $1 million. This letter must be sent to the bank of the exporter before the exporter will actually ship the goods. When the exporter delivers the shipping documents to her bank, she will receive the present value of the $1 million, discounted because the payment will not be made for 45 days.

The final step in the creation of a bankers acceptance is that the exporter's bank presents the evidence of shipment to the issuing bank (the importer's bank) which then "accepts" the evidence of shipment. It is this accepted promise to pay $1 million in 45 days that is the bankers acceptance. The importer will sign documents evidencing his obligation to his bank and becomes the borrower of the funds. When this final step is completed, the importer receives the documents necessary to receive the shipment of goods.

The exporter's bank can either continue to hold the acceptance or sell it to an investor, often a money market fund interested in short-term paper. The acceptance is a discount instrument and sells for the present value of the single $1 million payment to be made 45 days from the shipping date. The secondary market for bankers acceptances is limited so their liquidity is limited and most purchasers intend to hold them until their maturity dates.

The credit risk of a bankers acceptance is the risk that the importer (the initial borrower of the funds) and the accepting bank will both fail to make the promised payment.

LOS 64.j: Define an asset-backed security, describe the role of a special purpose vehicle in an asset-backed securities transaction, state the motivation for a corporation to issue an asset-backed security, and describe the types of external credit enhancements for asset-backed securities.

Credit card debt, auto loans, bank loans, and corporate receivables are often securitized in the same way as mortgages are in the MBS structure. These financial assets are the underlying collateral for bonds which are also **asset-backed securities** (ABSs). While the above types of underlying assets are the most common, innovative ABSs have also been created. In one case, singer David Bowie sold a $55 million dollar ABS issue where the underlying assets were the royalties from 25 of his albums released prior to 1990.

Role of a Special Purpose Vehicle

A **special purpose vehicle** or *special purpose corporation* is a separate legal entity to which a corporation transfers the financial assets for an ABS issue. The importance of this is that a legal transfer of the assets is made to the special purpose vehicle. This shields the assets from the claims of the corporation's general creditors, making it possible for the ABS issue to receive a higher credit rating than the corporation as a whole. Because the assets are sold to the special purpose vehicle, they are highly unlikely to be subject to any claims arising from the bankruptcy of the corporation, and the special purpose vehicle is termed a *bankruptcy remote* entity.

The **motivation for a corporation to issue asset-backed securities** is to reduce borrowing costs. By transferring the assets into a separate entity, the entity can issue the bonds and receive a higher rating than the unsecured debt of the corporation. The higher rating reduces the required yield on the (ABS) debt.

External Credit Enhancements

Since asset-backed securities, on their own, may not receive the highest possible credit rating, the issuer may choose to enhance the credit rating by providing additional guarantees or security. Credit quality can be enhanced either externally or internally. **External credit enhancement** commonly takes the following forms:

- *Corporate guarantees*, which may be provided by the corporation creating the ABS or its parent.
- *Letters of credit*, which may be obtained from a bank for a fee.
- *Bond insurance*, which may be obtained from an insurance company or a provider specializing in underwriting such structures. This is also referred to as an *insurance wrap*.

None of these enhancements come without cost. The decision of how much enhancement to provide involves a tradeoff between the cost of enhancement and the resulting decrease in the market yield required on the bonds.

Note that the quality of a credit-enhanced security is only as good as the quality of the guarantor, and the credit rating of the security can reflect any deterioration in the guarantor's rating.

LOS 64.k: Describe collateralized debt obligations.

A **collateralized debt obligation** (CDO) is a debt instrument where the collateral for the promise to pay is an underlying pool of other debt obligations and even other CDOs. These underlying debt obligations can be business loans, mortgages, debt of developing countries, corporate bonds of various ratings, asset-backed securities, or even problem/non-performing loans. Tranches of the CDO are created based on the seniority of the claims to the cash flows of the underlying assets, and these are given separate credit ratings depending on the seniority of the claim as well as the creditworthiness of the underlying pool of debt securities.

CDOs may be created by a sponsor that seeks to profit on the spread between the rate to be earned on the underlying assets and the rate promised to the CDO holder (an arbitrage CDO) or created by a bank or insurance company seeking to reduce its loan exposure on its balance sheet (a balance sheet CDO).

LOS 64.l: Contrast the structures of the primary and secondary markets in bonds.

The **primary market** for debt (newly created debt securities) functions in a manner similar to the primary market for equities. Typically, an investment banker is involved in advising the debt issuer and in distributing (selling) the debt securities to investors. When the investment banker actually purchases the entire issue and resells it, they are said to have "underwritten" the issue. This arrangement is termed a *firm commitment* and the deal, a *bought deal.* In an underwritten offering of debt securities, the underwriter will typically put together a syndicate of other investment bankers to aid in distributing the securities. The underwriters can reduce their risk by pre-selling as much of the offering as possible to their institutional clients and hedging the interest rate risk exposure of the issue for the period they anticipate owning the securities. An alternative is for the investment banker to agree to sell all of the issue that they can and this is termed doing the offering on a *best efforts* basis.

In the above described process, since the price paid for the issue and the anticipated sale price is determined between the (lead) investment bank and the issuing company, the offering is termed a *negotiated offering.* Another approach is an *auction process* where an issuer of debt securities determines the size and terms of the issue and several investment banks, or underwriting syndicates of multiple investment banks, bid on what interest rate they require to sell it. The syndicate with the lowest interest rate bid will be awarded the deal.

In the U.S., securities to be offered to public investors must be registered with the SEC. When a new issue of debt securities is not registered for sale to the public, it still may be sold to a small number of investors. This is called a *private placement* or Rule 144A offering (after the rule that allows such transactions). Avoidance of the registration process is valuable to the issuer and, since a private placement involves a sale to a small number of investors/institutions, the issue can be tailored to the needs and preferences of the buyers. Since the issue cannot be sold to the public unless it is subsequently registered, the buyers will require a slightly higher interest rate to compensate them for the lack of liquidity of securities that are sold though a private placement.

The **secondary market** for debt securities includes exchanges, an over-the-counter dealer market, and electronic trading networks. Traditionally, most secondary trading in debt securities was transacted in a dealer market, with broker/dealers buying and selling bonds for and from their inventories (i.e., acting as market makers). More recently, the costs and risks of supplying the capital necessary to adequately fund bond trading operations have increased and spreads have decreased. Because of this, electronic trading has become a more important part of the secondary market for debt securities. These electronic networks can be bids and offers by a single dealer, bids and offers by multiple dealers, or simply anonymous customer bids and offers posted on an electronic trading system with a trade clearing system.

KEY CONCEPTS

1. Asset-backed securities are debt that is supported by an underlying pool of mortgages, auto loans, credit card receivables, commercial loans, or other financial assets.
   - Foreign bonds are issued and traded in one country by a firm that is a non-resident in that country and are usually regulated by the authorities of the country in which they are issued.
   - Eurobonds are issued outside the legal system of any one country, are offered simultaneously to many investors internationally, are unregistered, are typically unsecured, are offered by issuers with high credit ratings, and are underwritten by large international syndicates.
   - Global bonds trade in U.S. markets as well as in the Eurobond market and have the following characteristics: (1) issuers must have consistent borrowing needs for periodic trips to the market, (2) funds must be needed in large amounts, and (3) the issuer must have a high credit rating.
2. Sovereign debt refers to the debt obligations of governments. U.S. Treasury securities are sovereign debt.
3. The primary securities issued by the U.S. Treasury are bills (pure-discount securities maturing in four weeks, three months, or six months), notes (maturing in two to ten years), and bonds (maturing in more than ten years).
4. Treasury Inflation Protected Securities (TIPS) are U.S. Treasury issues in which the coupon rate is fixed but the par value is adjusted periodically to account for inflation.
5. Treasury strips are traded in two forms—coupon strips and principal strips—and are taxed by the IRS on the basis of accrued interest, like other zero-coupon securities.
6. U.S. Treasuries trade in the secondary market and are termed on-the-run (i.e., the most recent issue) or off-the-run (i.e., older issues).
7. Prices for U.S. Treasuries are quoted in whole percent and 32nds of a percent of par value; a price of 92-11 is a price of 92 and 11/32nds percent of 1,000 or $923.44.
8. A mortgage-backed security (MBS) is backed by a pool of amortizing mortgage loans (the collateral) and has monthly cash flows that include both interest and principal payments.
9. Prepayment risk is significant for investors of MBSs, since most mortgage loans contain a prepayment option.
10. State and local government securities (municipal securities or munis) are usually exempt from U.S. federal taxes, and state-tax exempt in the state of issuance.
11. Municipal bonds include tax-backed debt, backed by the taxing authority of the governmental unit issuing the securities, and revenue bonds, backed only by the revenues from the project financed by the bond issue.
12. Insured bonds are guaranteed by a firm specializing in bond insurance and prerefunded bonds are back by U.S. Treasury securities.
13. Corporate securities include bonds, medium-term notes, and commercial paper.
14. Bond rating agencies rate corporate bonds on capacity to repay (liquid assets and cash flow), management quality, industry prospects, corporate strategy, financial policies, credit history, overall debt levels, the collateral for the issue, and the nature of the covenants.
15. Corporate bonds may be secured or unsecured (called debentures). Security can be in the form of real property, financial assets, or personal property/equipment.
16. Medium-term notes (MTN) are issued periodically by corporations under a shelf registration, sold by agents on a best-efforts basis, and have maturities ranging from 9 months to over 30 years.
17. Commercial paper is a short-term corporate financing vehicle and does not require registration with the SEC if its maturity is less than 270 days. CP comes in two forms: (1) directly-placed paper—sold directly by the issuer; and (2) dealer-placed paper—sold to investors through agents/brokers.
18. Negotiable CDs are issued in a wide range of maturities by banks, trade in a secondary market, are backed by bank assets, and are termed Eurodollar CDs when denominated in $U.S. and issued outside the U.S.
19. Bankers acceptances are issued by banks to guarantee a future payment for goods shipped, sold at a discount to the future payment they promise, short-term, and have limited liquidity.

20. Collateralized debt obligations (CDOs) are backed by an underlying pool of debt securities which may be any one of a number of types: corporate bonds, loans, emerging markets debt, mortgage-backed securities, or other CDOs.
21. The primary market in bonds includes underwritten and best-efforts public offerings, as well as private placements.
22. The secondary market in bonds includes some trading on exchanges and a much larger volume of trading in a dealer (OTC) market, but electronic trading networks continue to be an increasingly important part of the secondary market for bonds.

Exam Flashbacks

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #116 from '88 actual exam.

The term foreign bond market refers to trading in:
A. issues sold in one country in a currency of another country by a borrower from another country.
B. issues sold in the borrower's country in a currency of another country.
C. issues sold in one country and currency by a borrower from another country.
D. none of the above.

Exam Flashback # 2

Source: Question #67 from '91 actual exam.

Yankee bonds are U.S.-pay bonds that are:
A. traded principally in Japan and the U.K.
B. issued by foreign-domiciled issuers who register with the SEC.
C. free of withholding tax to non-U.S. investors.
D. all of the above.

Exam Flashback # 3

Source: Question #104 from '99 sample exam.

The following are quotes for a U.S. Treasury bond:

Bid	Ask
102:2	102:5

If the face value of the bond is $1,000, the price an investor should pay for the bond is *closest* to:
A. $1,020.63.
B. $1,021.56.
C. $1,025.00.
D. $1,026.25.

Exam Flashback # 4

Source: Question #23 from '90 actual exam.

A revenue bond is distinguished from a general obligation bond in that revenue bonds:
A. are issued by counties, special districts, cities, towns and state-controlled authorities, whereas general obligation bonds are only issued by the states themselves.
B. are typically secured by limited taxing power, whereas general obligation bonds are secured by unlimited taxing power.
C. are issued to finance projects and are secured by the revenues of the project being financed.
D. have first claim to any revenue increase of the tax authority issuing the bonds.

Concept Checkers: Overview of Bond Sectors and Instruments

1. A Treasury security is quoted at 97-17 and has a par value of $100,000. Which of the following is its quoted dollar price?
   A. $97,170.00.
   B. $97,531.25.
   C. $100,000.00.
   D. $975,312.50.

2. An investor holds $100,000 (par value) worth of Treasury Inflation Protected Securities (TIPS) that carry a 2.5 percent semiannual pay coupon. If the annual inflation rate is 3 percent, what is the inflation-adjusted principal value of the bond after six months?
   A. $100,000.
   B. $101,500.
   C. $102,500.
   D. $103,000.

3. An investor holds $100,000 (par value) worth of TIPS currently trading at par. The coupon rate of 4 percent is paid semiannually, and the annual inflation rate is 2.5 percent. What coupon payment will the investor receive at the end of the first six months?
   A. $2,000.
   B. $2,025.
   C. $2,050.
   D. $4,000.

4. A Treasury note (T-note) principal strip has six months remaining to maturity. How is its price likely to compare to a 6-month Treasury bill (T-bill) that has just been issued? The T-note price should be:
   A. lower.
   B. higher.
   C. the same.
   D. set at the coupon rate.

5. Which of the following statements is *most correct*?
   A. Treasury principal strips are usually created from Treasury bills.
   B. Treasury bonds may be used to create Treasury coupon STRIPS.
   C. Treasury coupon strips can be created from Treasury bonds, notes, or bills.
   D. Treasury coupon strips make lower coupon payments than Treasury principal strips.

6. Which of the following municipal bonds typically has the *greater* risk and is issued with *higher* yields?
   A. Revenue bonds.
   B. Appropriation backed obligations.
   C. Limited tax general obligation bonds.
   D. Unlimited tax general obligation bonds.

7. A bond issue that is serviced with the earnings from a pool of Treasury securities that have been placed in escrow is called a(n):
   A. insured bond.
   B. prerefunded bond.
   C. absolute priority bond.
   D. credit-enhanced obligation.

8. Of the following, the debt securities that are most often registered according to the requirements of SEC Rule 415 (shelf registration) are:
   A. corporate bonds.
   B. medium-term notes.
   C. double-barreled bonds.
   D. mortgage-backed securities.

9. A Yankee bond and a Samurai bond are bonds that are issued and traded in the United States and Japan by issuers that are domiciled in a country other than the U.S. or Japan, respectively. These bonds are referred to as:
   A. eurobonds.
   B. foreign bonds.
   C. sovereign debt.
   D. special purpose vehicles.

10. A corporation issuing asset-backed securities can often improve the credit rating of the securities to above that of the issuing company by transferring the assets to a(n):
    A. asset trust.
    B. special purpose vehicle.
    C. bond insurer.
    D. fiduciary account.

11. Which of the following is a *difference* between an on-the-run and an off-the-run issue? An on-the-run issue:
    A. will always carry a higher coupon.
    B. is the most recently issued security of that type.
    C. has a shorter maturity than an off-the-run issue.
    D. is publicly traded whereas an off-the-run issue is not.

12. Compared to a public offering, a private placement of debt securities *likely* has:
    A. more liquidity and a lower yield.
    B. more liquidity and a higher yield.
    C. less liquidity and a lower yield.
    D. less liquidity and a higher yield.

13. Compared to negotiable CDs, bankers acceptances:
    A. are more liquid.
    B. are less likely to default.
    C. have shorter maturities on average.
    D. are more likely to pay periodic interest.

14. A debt security that is collateralized by a pool of the sovereign debt of several developing countries is *likely* a(n):
    A. CMO.
    B. CDO.
    C. EMD.
    D. ABS.

15. Activities in the primary market for debt securities would **NOT** include:
    A. market making.
    B. an auction process.
    C. a best-efforts offering.
    D. a firm commitment.

ANSWERS – EXAM FLASHBACKS

1. **C** Examples of foreign bonds are Yankee bonds (denominated in dollars, trade on U.S. exchanges, and issued by a non-U.S. company), Samurai bonds (denominated in yen, trade on Japanese exchanges, and issued by a non-Japanese company), and Bulldog bonds (denominated in pounds, trade on a U.K. exchange, and issued by a non-U.K company).

2. **B** CFA Institute rarely asks questions that include "all of the above" or "none of the above" as one of the choices at Level 1. On the exam, you should watch out for four viable choices. Yankee Bonds are issued by foreign entities and trade on U.S. exchanges. As such, they are subject to SEC registration requirements.

3. **B** This question is looking for two things:

   (1) Treasuries are quoted in 32nds: $\frac{5}{32} = 0.15625$ and

   (2) when you buy you pay the ask, when you sell you get the bid. Hence, the price is 102.15625 per hundred or \$1,021.56.

4. **C** Notice in answer "A" that the word "only" is used. This word should be a red flag in any question. Other red flags include "never" and "always." Revenue bonds are not secured by the general taxing authority of the issuing governmental body and are issued to finance specific projects. The revenue from the project is what is used to repay the bond issue.

ANSWERS – CONCEPT CHECKERS: OVERVIEW OF BOND SECTORS AND INSTRUMENTS

1. **B** This value is computed as follows: dollar price = $97\frac{17}{32}\% \times \$100{,}000 = 0.9753125 \times \$100{,}000 = \$97{,}531.25$.

2. **B** The annual inflation rate is 3%, which corresponds to 1.5% semiannually. Therefore, the principal value has increased by 1.5%. So we have: new principal = \$100,000 × 1.015 = \$101,500.

3. **B** This coupon payment is computed as follows:

   $$\text{Coupon payment} = (\$100{,}000 \times 1.0125)\left(\frac{0.04}{2}\right) = \$2{,}025$$

4. **C** The T-note principal strip has exactly the same cash flows (the principal) as the T-bill. Therefore, the prices of the two securities should be (about) equal. However, market imperfections, such as illiquidity, may lead to differences.

5. **B** Treasury coupon and principal strips are created by separating (stripping) the principal and coupons from Treasury notes and bonds and selling packages of these single-maturity cash flows as individual zero-coupon securities. Treasury bills cannot be used because they are already zero-coupon securities.

6. **A** Revenue bond issues are only obligated to pay principal and interest if revenue from the project that they helped fund is sufficient enough to service the issue. When issued, revenue bonds typically are riskier than general obligation bonds and, consequently, have higher yields.

7. **B** The cash flows generated by an escrow pool of Treasury securities are used to service prerefunded bonds. Insured bonds carry third-party guarantees. There are no securities formally known as absolute priority bonds or credit enhanced obligations (yet).

8. **B** Shelf registration is used with medium-term notes. This permits the issue to be held in inventory (on the "shelf") and sold in parcels at the discretion of the issuer. Corporate, MBS, and double-barreled municipal bond issues are usually sold all at once.

9. **B** Foreign bonds are traded and issued in a country other than the country of the issuer. For example, the bonds of Toyota Motor Company, a Japanese firm, issued and traded in Great Britain are foreign bonds (called bulldog bonds).

10. **B** The assets are sold to a special purpose vehicle to protect them from general claims against the issuing corporation.

11. **B** On-the-run issues are the most recently issued securities.

12. **D** Investors require a higher yield to compensate for the fact that privately placed debt is not registered for public sale and is therefore less liquid than debt registered for public sale.

13. **C** Bankers acceptances are short-term and pay no periodic interest. Like negotiable CDs, they are as good as the credit of the issuing bank but have a very limited secondary market.

14. **B** A CDO or collateralized debt obligation is backed by an underlying pool of debt securities which may be emerging markets debt. A CMO is backed by a pool of mortgages, an ABS is backed by financial assets, and EMD is made up.

15 **A** Market making refers to a dealer that trades in the secondary market for its own account from inventory.

The following is a review of the Debt Investments: Basic Concepts principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

UNDERSTANDING YIELD SPREADS

Study Session 14

EXAM FOCUS

Yield spreads are simply differences between the yields of any two debt securities or types of debt securities. Try to get a good grip on the spread terminology in this review and the characteristics that drive yield spreads. Theories of the term structure have been moved to this topic review this year. You should know all three theories, not only their implications for the shape of the yield curve but also what the yield curve shape can tell us under each of the three theories. Learn the relationships between taxable and after-tax yields and between tax-free and taxable equivalent yields well. Calculations of these relations are almost sure to be worth some points come exam day.

LOS 65.a: Identify the interest rate policy tools available to a central bank (such as the U.S. Federal Reserve or the European Central Bank).

While interest rates are determined by a variety of economic conditions, in the U.S. the Federal Reserve (Fed) attempts to manage short-term rates through its *monetary policy tools*. The **four interest rate tools of the Fed** are as follows:

1. **The discount rate** is the rate at which banks can borrow reserves from the Fed. A lower rate tends to increase bank reserves, encourage lending, and decrease interest rates. A higher discount rate has the opposite effect, raising rates.

2. **Open market operations** refers to the buying or selling of Treasury securities by the Fed in the open market. When the Fed buys securities, cash replaces securities in investor accounts, more funds are available for lending, and interest rates decrease. Sales of securities by the Fed have the opposite effect, reducing cash balances and funds available for lending as well as increasing rates.

3. **Bank reserve requirements** are the percentage of deposits that banks must retain (not loan out). By increasing the percentage of deposits banks are required to retain as *reserves,* the Fed effectively decreases the funds that are available for lending. This decrease in amounts available for lending will tend to increase interest rates. A decrease in the percentage reserve requirement will increase the funds available for loans and tends to decrease interest rates.

4. **Persuading banks to tighten or loosen their credit policies.** By asking banks to alter their lending policies, the Fed attempts to affect their willingness to lend. Encouraging lending will tend to decrease rates and vice versa.

The most commonly used policy tool is *open market operations.*

LOS 65.b: Describe a yield curve and the different yield curve shapes observed and explain the basic theories of the term structure of interest rates (i.e., pure expectations theory, liquidity preference theory, and market segmentation theory) and describe the implications of each theory for the shape of the yield curve; explain the different types of yield spread measures (e.g., absolute yield spread, relative yield spread, yield ratio), and compute yield spread measures given the yields for two securities.

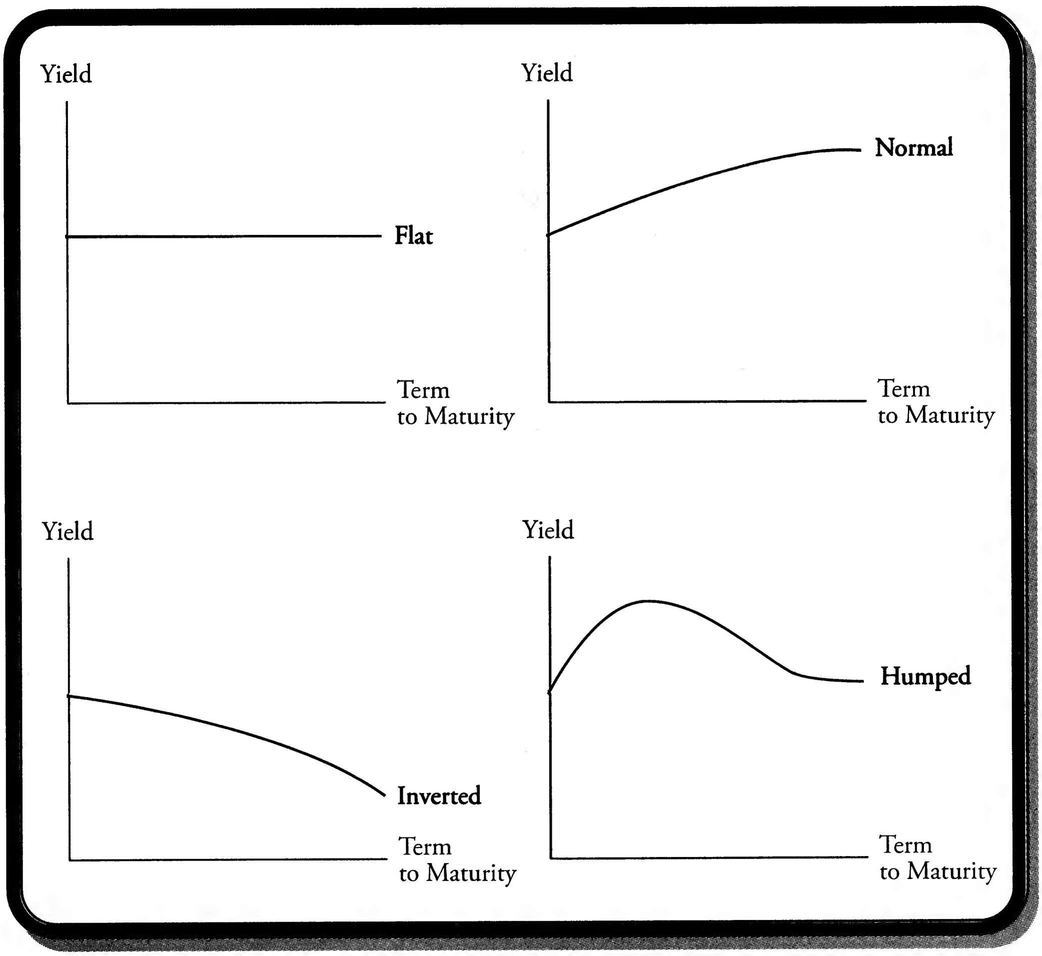
We have mentioned yield curves previously as just a plot of yields by years to maturity. For a view of some current yield curves and related information, you can look at www.bloomberg.com/markets/rates/index.html.

There are four general shapes that we use to describe yield curves:

- Normal or upward sloping.
- Inverted or downward sloping.
- Flat.
- Humped.

These four shapes are illustrated in Figure 1.

Figure 1: Yield Curve Shapes



Yield curves can take on just about any shape, so don't think these examples are the only ones observed. These four are representative of general types, and you need to be familiar with what is meant by an 'upward sloping' or "normal" yield curve and by an "inverted" or "downward sloping" yield curve. Humped and flat yield curves

usually go by just those descriptive names and shouldn't present any problem. Just remember that a flat yield curve means that yields are all equal at every maturity.

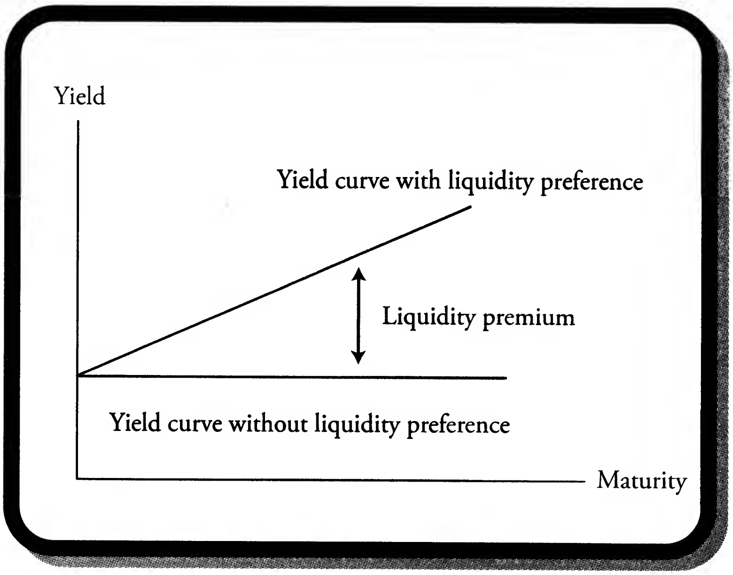
Theories of the Term Structure of Interest Rates

The **pure expectations theory** states that the yield for a particular maturity is an average (not a simple average) of the short term rates that are expected in the future. If short term rates are expected to rise in the future, interest rate yields on longer maturities will be higher than those on shorter maturities, and the yield curve will be upward sloping. If short term rates are expected to fall over time, longer maturity bonds will be offered at lower yields.

Proponents of the **liquidity preference theory** believe that, in addition to expectations about future short term rates, investors require a risk premium for holding longer term bonds. This is consistent with the fact that interest rate risk is greater for longer maturity bonds.

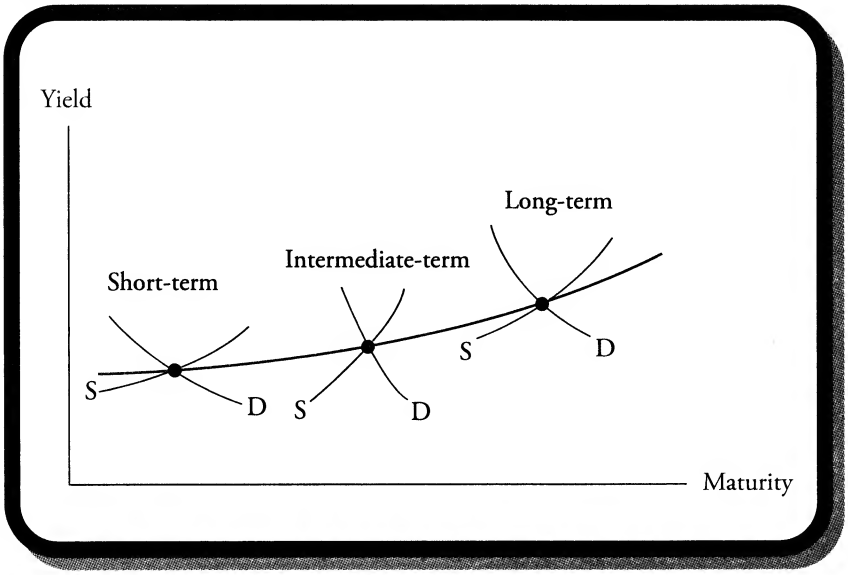
Under this theory, the size of the liquidity premium will depend on how much additional compensation investors require to induce them to take on the greater risk of longer maturity bonds or, alternatively, how strong their preference for the greater liquidity of shorter term debt is. An illustration of the effect of a liquidity premium on a yield curve, where expected future short-term rates are constant, is presented in Figure 2.

Figure 2: Liquidity Premium



The **market segmentation theory** is based on the idea that investors and borrowers have preferences for different *maturity ranges*. Under this theory, the supply of bonds (desire to borrow) and the demand for bonds (desire to lend) determine equilibrium yields for the various maturity ranges. Institutional investors may have strong preferences for maturity ranges that closely match their liabilities. Life insurers and pension funds may prefer long maturities due to the long-term nature of the liabilities they must fund. A commercial bank that has liabilities of a relatively short maturity may prefer to invest in shorter-term debt securities. Another argument for the market segmentation theory is that there are legal or institutional policy restrictions that prevent investors from purchasing securities with maturities outside a particular maturity range. The determination of yields for various maturity ranges of the yield curve is illustrated in Figure 3.

Figure 3: Market Segmentation Theory and the Yield Curve



A somewhat weaker version of the market segmentation theory is the *preferred habitat theory.* Under this theory, yields also depend on supply and demand for various maturity ranges, but investors can be induced to move from their preferred maturity ranges when yields are sufficiently higher in other (non-preferred) maturity ranges.

The Shape of the Yield Curve

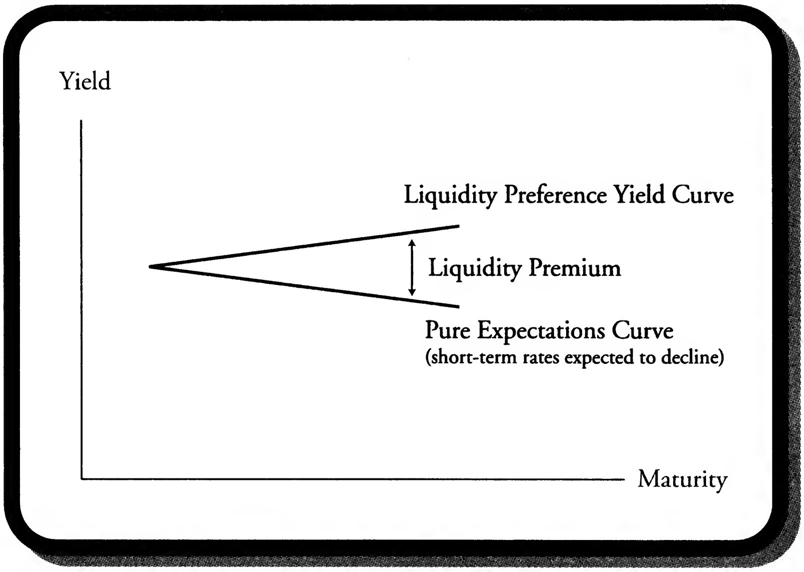
The **pure expectations theory** by itself has no implications for the shape of the yield curve. The various expectations and the shapes that are consistent with them are:

Short term rates expected to rise in the future → upward-sloping yield curve
Short term rates are expected fall in the future → downward-sloping yield curve
Short term rates expected to rise then fall → humped yield curve
Short term rates expected to remain constant → flat yield curve

The shape of the yield curve, under the pure expectations theory, provides us with information about investor expectations about future short-term rates.

Under the **liquidity preference theory,** the yield curve may take on any of the shapes we have identified. If rates are expected to fall a great deal in the future, even adding a liquidity premium to the resulting negatively sloped yield curve can result in a downward sloping yield curve. A humped yield curve could still be humped even with a liquidity premium added to all the yields. Also note that, under the liquidity preference theory, an upward sloping yield curve can be consistent with expectations of declining short term rates in the future. This case is illustrated in Figure 4.

Figure 4: Liquidity Premium Added to Decreasing Expected Rates



The **market segmentation theory** of the term structure is consistent with any yield curve shape. Under this theory, it is supply and demand for debt securities at each maturity range that determines the yield for that maturity range. There is no specific linkage among the yields at different maturities, although, under the *preferred habitat theory*, higher rates at an adjacent maturity range can induce investors to purchase bonds with maturities outside their preferred range of maturities.

Types of Yield Spread Measures

A yield spread is simply the difference between the yields on two bonds or two types of bonds. Three different yield spread measures are as follows:

The **absolute yield spread** is simply the difference between yields on two bonds. This simple measure is sometimes called the *nominal spread*. Absolute yield spreads are usually expressed in basis points (100ths of 1 percent).

$$\text{absolute yield spread} = \text{yield on the higher-yield bond} - \text{yield on the lower-yield bond}$$

The **relative yield spread** is the absolute yield spread expressed as a percentage of the yield on the lower-yield bond.

$$\text{relative yield spread} = \frac{\text{absolute yield spread}}{\text{yield on the lower-yield bond}} = \frac{\text{higher yield}}{\text{lower yield}} - 1$$

The **yield ratio** is the ratio of the yields on the two bonds:

$$\text{yield ratio} = \frac{\text{higher yield}}{\text{lower yield}}$$

Note that the yield ratio is simply one plus the relative yield spread. The calculation of these yield spread measures is illustrated in the following example.

Example: Computing yield spreads

Consider two bonds, X and Y. Their respective yields are 6.50 percent and 6.75 percent. Using bond X as the reference bond, **compute** the absolute yield spread, the relative yield spread, and the yield ratio for these bonds.

Answer:

Absolute yield spread = 6.75% – 6.50% = 0.25% or 25 basis points
Relative yield spread = 0.25% / 6.50% = 0.038 = 3.8%
Yield ratio = 6.75% / 6.50% = 1.038

LOS 65.c: Explain why investors may find a relative yield spread to be a better measure of yield spread than the absolute yield spread, distinguish between an intermarket and intramarket sector spread, and describe a credit spread and discuss the suggested relationship between credit spreads and the economic well being of the economy.

The most commonly used yield spread is the ***absolute*** yield spread, even though it is the most simplistic. A shortcoming of the absolute yield spread is that it may remain constant, even though overall rates rise or fall. In this case, the effect of rising or falling rates on spreads is captured by the relative yield spread or the yield ratio.

For example, consider two yields that rise from 6.5% and 7% to 7% and 7.5%, respectively. The absolute yield spread remains constant at 50 basis points, while the relative spread falls from 7.69% to 7.14% and the yield ratio decreases from 1.077 to 1.071.

Intermarket and Intramarket Sector Spreads

The U.S. bond market is segmented into different *sectors* on the *basis of the bond issuer*. The major U.S. bond market sectors are:

- U.S. government sector.
- U.S. government agencies sector.
- Municipal sector.
- Corporate sector (includes industrials, utilities, financials, banking).
- Mortgage sector.
- Asset-based securities sector.
- Foreign sector (includes sovereign debt, corporates, and supranationals).

An **intermarket sector yield spread** refers to differences between the yields of a bond in one sector and one in a different sector. The most commonly quoted intermarket spread is the spread between a corporate bond (e.g., the AT&T bonds maturing in 2013) and the yield on a similar-maturity Treasury bond. These spreads will be different at different maturities and tend to increase with maturity. Any yield spread between bonds in two different sectors or between the average yields of types of bonds in two different sectors is an intermarket sector spread.

An **intramarket sector spread** refers to the yield spread between two bonds or two types of bonds belonging to the same market sector. For example, long Treasuries versus short Treasuries (*a maturity spread)* and AA-rated corporates versus BBB-rated corporates (*a credit or quality spread*) are both intramarket sector spreads.

Credit Spreads

A **credit (or quality) spread** is the difference in yields between two issues that are similar in all respects except for credit rating. An example of a credit spread is the difference in yields between long AA-rated general obligation (GO) municipal bonds and long A-rated GO munis (an intramarket spread as well). Obviously, these spreads

show the effect of credit quality on yields and reveal the risk-return tradeoff the investor can expect (i.e., how much added return an investor can earn by investing in issues with higher perceived credit risk).

Credit spreads are related to the state of the economy. During an expanding economy, credit spreads decline as corporations are expected to have stronger cash flows. On the other hand, during economic contractions, cash flows are pressured, leading to a greater probability of default and higher yields on lower-quality issues. When investors anticipate an economic downturn, they often sell low-quality issues and buy high-quality issues, including Treasuries. This "flight to quality" puts downward pressure on the prices of low quality issues, raising their yields.

LOS 65.d: Identify how embedded options affect yield spreads.

A call option on a bond is an option the bond issuer holds and will only be exercised if it is advantageous to the issuer to do so. From the bondholder's perspective, a non-callable bond is preferred to a bond that is otherwise identical but callable. Investors will require a higher yield on a callable bond, compared to the same bond without the call feature. Therefore, yield spreads to a benchmark bond, such as a similar maturity Treasury issue, are higher for the callable bond. By the same reasoning, yield spreads must be greater to compensate bondholders for the prepayment option embedded in mortgage passthrough securities.

The inclusion of a put provision or a conversion option with a bond will have the opposite effect; the choice of whether to exercise either of these options is the bondholder's. Compared to an identical option-free bond, a putable bond will have a lower yield spread to Treasuries due to the value of the put feature 'included' with the bond.

The fact that option provisions affect yield spreads is important because this tells us that spreads for bonds with embedded options are not purely premiums for credit risk, liquidity differences, and maturity (duration) risk.

LOS 65.e: Explain how the liquidity of an issue affects its yield spread relative to Treasury securities and relative to other issues that are comparable in all other ways except for liquidity and describe the relationships that are argued to exist among the size of an issue, liquidity, and yield spread.

Bonds that have **less liquidity have higher spreads** to Treasuries. Investors prefer more liquidity to less and will pay a premium for greater liquidity. A higher price for a bond that is identical to another in all aspects except that it is more actively traded—and therefore more liquid—translates into a lower yield compared to the less liquid bond.

Liquidity is affected by the size of an issue. **Larger issues normally have greater liquidity** because they are more actively traded in the secondary market. Empirical evidence suggests that issues with **greater size have lower yield spreads.** When compared with identical but smaller issues, larger-size issues have lower yields due to their greater liquidity.

LOS 65.f: Compute the after-tax yield of a taxable security and the tax-equivalent yield of a tax-exempt security.

The **after-tax yield** on a taxable security can be calculated as:

$$\text{after-tax yield} = \text{taxable yield} \times (1 - \text{marginal tax rate})$$

Example: Computing after-tax yield

What is the after-tax yield on a corporate bond with a yield of 10% for an investor with a 40% marginal tax rate?

Answer:

Investors are concerned with after-tax returns. The marginal tax rate is the percentage that must be paid in taxes on one additional dollar of income, in this case interest income.

For an investor with a marginal tax rate of 40 percent, 40 cents of every additional dollar of taxable interest income must be paid in taxes. For a taxable bond that yields 10 percent, the after-tax yield to an investor with a 40 percent marginal tax rate will be:

$$10\%(1 - 0.4) = 6.0\% \text{ after tax}$$

Tax-exempt securities can offer lower yields compared to taxable securities because the yields they offer are after-tax yields. The higher an investor's marginal tax rate, the greater the attractiveness of a tax exempt issue compared to a taxable issue. The taxable-equivalent yield is the yield a particular investor must earn on a taxable bond to have the same after-tax return they would receive from a particular tax-exempt issue. The calculation is just a rearrangement of the after-tax yield formula above.

$$\text{taxable-equivalent yield} = \frac{\text{tax-free yield}}{(1 - \text{marginal tax rate})}$$

Example: Taxable-equivalent yield

Consider a municipal bond that offers a yield of 4.5 percent. If an investor is considering buying a fully taxable Treasury security offering a 6.75 percent yield, should she buy the Treasury security or the municipal bond, given that her marginal tax rate is 35 percent?

Answer:

We can approach this problem from two perspectives. First, the taxable equivalent yield on the municipal bond is $\frac{4.5\%}{(1-0.35)} = 6.92\%$, which is higher than the taxable yield, so the municipal bond is preferred.

Alternatively, the after-tax return on the taxable bond is $0.0675 \times (1 - 0.35) = 4.39\%$.

Thus, the after-tax return on the municipal bond (4.5 percent) is greater than the after-tax yield on the taxable bond (4.39 percent), and the municipal bond is preferred.

Either approach gives the same answer; she should buy the municipal bond. (*See Exam Flashbacks #1 and #2.*)

LOS 65.g: Define LIBOR and why it is an important measure to funded investors who borrow short-term.

We previously mentioned **LIBOR** (London Interbank Offered Rate) in reference to the rates paid on negotiable CDs by banks and bank branches located in London. LIBOR has become the most important benchmark or reference rate for floating-rate debt securities and short-term lending. LIBOR is determined each day and published by the British Bankers' Association for several currencies, including the U.S., Canadian, and Australian dollars, the Euro, Japanese yen, British pounds, and Swiss francs, among others. While the maturity of the CDs

that banks invest in can range from overnight to five years, LIBOR is most important for short-term rates of 1 year or less.

A **funded investor** is one who borrows to finance an investment position. The importance of LIBOR in this context is as a measure of the funding costs because the loans to finance the investment are most often floating-rate loans or short-term loans where the reference rate is published LIBOR. Recall that floating-rate loans are based on a reference rate plus a margin. A funded investor with a borrowing rate of 2-month (60-day) LIBOR + 40 basis points would have a borrowing cost (annualized) of 2.60 percent when 2-month LIBOR is quoted at 2.2 percent. The profits of such a funded investor would depend on his or her ability to earn greater than a 2.6 percent annual rate on the investments funded in such a manner.

KEY CONCEPTS

1. A central bank's tools for affecting short-term interest rates are the discount rate, open-market operations, the reserve requirement, and persuasion to influence banks lending policies.
2. The "yield curves" represent the plot of yield against maturity for a particular type of bonds. The general shapes are upward or downward sloping, flat, or humped.
3. Theories of the yield curve and their implications for the shape of the yield curve are:
   - The pure expectations theory argues that rates at longer maturities depend only on expectations of future short-term rates and is consistent with any yield curve shape.
   - The liquidity preference theory of the term structure states that longer term rates reflect investors' expectations about future short-term rates as well as a liquidity premium to compensate them for exposure to interest rate risk. The liquidity preference theory adds an increasing term premium to all yield curve points but can be consistent with a downward sloping curve if an expected decrease in short-term rates outweighs the term premium.
   - The market segmentation theory argues that lenders and borrowers have preferred maturity ranges and that the shape of the yield curve is determined by the supply and demand for securities within each maturity range, independent of the yield in other maturity ranges. It is consistent with any yield curve shape and in a somewhat weaker form is known as the preferred habitat theory.
4. Types of yield spreads:
   - The absolute yield spread is the difference between the yield on a particular security or sector and the yield of a reference (benchmark) security or sector, which is often on-the-run Treasury securities of like maturity.
   - The relative yield spread is the absolute yield spread expressed as a percentage of the benchmark yield. This is arguably a superior measure to the absolute spread, since it will reflect changes in the level of interest rates even when the absolute spread remains constant.
   - The yield ratio is the ratio of the yield on a security or sector to the yield on a benchmark security or sector; it is simply one plus the relative yield spread.
5. An intermarket sector spread is the yield spread between comparable bonds in two different market sectors. Spreads between corporate bonds and Treasuries are intermarket sector spreads and tend to increase with maturity.
6. An intramarket spread is the difference in yields between two different issues in the same market sector. These spreads can be those due to differences in maturity, option provisions, credit rating, or liquidity.
7. A credit spread is the yield difference between two bond issues due to differences in credit (default) risk, as reflected in their credit ratings. These spreads narrow when the economy is healthy and expanding, while they increase during contractions/recessions reflecting a 'flight to (higher) quality' by investors.
8. Call options and pre-payment options increase yields and yield spreads compared to option-free bonds. Put options and conversion options decrease yields and yield spreads compared to comparable option-free bonds.
9. Empirical evidence suggests that larger debt issues by the same borrower are more liquid when other bond characteristics are alike. Greater liquidity implies a lower-liquidity risk premium, and more liquid bonds tend to have lower yields and lower yield spreads relative to a benchmark issue.

10. To compare a tax-exempt bond with a taxable issue, use either of the following:
    after-tax yield = taxable yield × (1 – marginal tax rate), and compare it to tax-exempt yield, or
    the taxable-equivalent yield = $\frac{\text{tax-free yield}}{(1 - \text{marginal tax rate})}$, and compare it to a taxable yield.

11. LIBOR is determined from rates on bank CDs in London in several important currencies and is the most important reference rate globally for floating-rate debt and short-term loans.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #95 from '94, '96 actual exams.

The coupon rate on a tax-exempt bond is 5.6 percent, and the rate on a taxable bond is 8 percent. Both bonds sell at par. The tax bracket (marginal tax rate) at which an investor would be indifferent between the two bonds is:
A. 30.0%.
B. 39.6%.
C. 41.7%.
D. 42.9%.

Exam Flashback # 2

Source: Question #83 from '92 actual exam

A municipal bond carries a coupon of 6 3/4% and is trading at par; to a taxpayer in the 34% tax bracket, this bond would provide a taxable equivalent yield of:
A. 4.5%.
B. 10.2%.
C. 13.4%.
D. 19.9%.

CONCEPT CHECKERS: UNDERSTANDING YIELD SPREADS

1. Under the pure expectations theory, an inverted yield curve is interpreted as evidence that:
   A. demand for long term bonds is falling.
   B. inflation is expected to rise in the future.
   C. short-term rates are expected to fall in the future.
   D. investors have very little demand for liquidity.

2. According to the liquidity preference theory, which of the following statements is FALSE?
   A. All else equal, investors prefer short-term securities over long-term securities.
   B. Long-term rates should be higher than short-term rates because of the added risks.
   C. Investors perceive little risk differential between short-term and long-term securities.
   D. Borrowers will pay a premium for long-term funds to avoid having to roll over short-term debt.

3. With respect to the term structure of interest rates, the market segmentation theory holds that:
   A. an increase in demand for long-term borrowings could lead to an inverted yield curve.
   B. expectations about the future of short-term interest rates are the major determinants of the shape of the yield curve.
   C. the yield curve reflects the maturity demands of financial institutions and investors.
   D. the shape of the yield curve is independent of the relationship between long- and short-term interest rates.

4. The most commonly used tool of the Fed to control interest rates is:
   A. the discount rate.
   B. the bank reserve requirement.
   C. open market operations.
   D. persuading banks to alter their lending policies.

5. For two bonds that are alike in all respects except maturity, the relative yield spread is 7.14 percent. The yield ratio is *closest* to:
   A. 92.85.
   B. 0.714.
   C. 1.0714.
   D. 107.14.

6. Assume the following yields for different bonds issued by a corporation:
   - One-year bond: 5.50%.
   - Two-year bond: 6.00%.
   - Three-year bond:7.00%.

   If the on-the-run 3-year U.S. Treasury is yielding 5 percent, then what is the ***absolute*** yield spread on the 3-year corporate issue?
   A. 0.40.
   B. 1.40.
   C. 100 bp.
   D. 200 bp.

7. Assume the following corporate yield curve:
   - One-year bond: 5.00%.
   - Two-year bond: 6.00%.
   - Three-year bond:7.00%.

   If an on-the-run 3-year U.S. Treasury is yielding 6 percent, the ***relative*** yield spread on the 3-year corporate is:
   A. 16.67%.
   B. 1.167.
   C. 14.28%.
   D. 100bp.

8. If a U.S. investor is forecasting that the yield spread between U.S. Treasury bonds and U.S. corporate bonds is going to widen, which of the following is *most likely* to be **TRUE**?
   A. The economy is going to expand.
   B. The economy is going to contract.
   C. There will be no change in the economy.
   D. The U.S. dollar will weaken.

9. For two bonds that are alike in all respects except credit risk, the yield ratio is 1.0833. If the yield on the higher yield bond is 6.5 percent, the lower yield bond yield is *closest* to:
   A. 8.33%.
   B. 5.50%.
   C. 7.04%.
   D. 6.00%.

10. Given two bonds that are equivalent in all respects except tax status, the marginal tax rate that will make an investor indifferent between an 8.2 percent taxable bond and a 6.2 percent tax-exempt bond is *closest* to:
    A. 24.39%.
    B. 76.61%
    C. 37.04%.
    D. 43.47%.

11. Which of the following statements *most accurately* describes the relationship between the economic health of a nation and credit spreads?
    A. Credit spreads and economic well-being are not correlated.
    B. Credit spreads decrease during an expanding economy because corporate cash flows are expected to rise.
    C. Credit spreads increase during an expanding economy because corporations invest in more speculative projects.
    D. Credit spreads increase during an expanding economy because corporations are expected to have volatile earnings.

12. Which of the following *most accurately* describes the relationship between liquidity and yield spreads relative to Treasury issues? All else being equal, bonds with:
    A. less liquidity have lower yield spreads to Treasuries.
    B. greater liquidity have higher yield spreads to Treasuries.
    C. less liquidity have higher yield spreads to Treasuries.
    D. greater liquidity have negative yield spreads to Treasuries.

13. A narrowing of credit spreads would *least* impact the value of which of the following investments?
    A. AAA corporate bond.
    B. 30-year Treasury bond.
    C. BB+ rated corporate bond.
    D. Callable corporate bond.

14. Assume an investor is in the 31 percent marginal tax bracket. She is considering the purchase of either a 7.5 percent corporate bond that is selling at par or a 5.25 percent tax-exempt municipal bond that is also selling at par. Given that the two bonds are comparable in all respects except their treatments, the investor should buy the:
    A. corporate bond, since it has the higher yield of 7.50%.
    B. municipal bond, since the taxable-equivalent yield on it is 10.87%.
    C. municipal bond, since its taxable-equivalent yield is 7.61%.
    D. corporate bond, since its after-tax yield is higher.

Answers – Exam Flashbacks

1. **A** taxable bond rate = $\dfrac{\text{municipal rate}}{1-\text{marginal tax rate}}$

   marginal tax rate = $1-\dfrac{\text{municipal rate}}{\text{taxable bond rate}}$

   $$\text{MTR} = 1-\frac{0.056}{0.08} = 0.30 \text{ or } 30\%$$

2. **B** taxable bond rate = $\dfrac{\text{municipal rate}}{(1-\text{marginal tax rate})}$

   $$\text{TBR} = \frac{0.0675}{1-0.34} = 0.102\text{, or } 10.2\%$$

Answers – Concept Checkers: Understanding Yield Spreads

1. **C** An inverted or downward-sloping yield curve, under the pure expectations theory, indicates that short-term rates are expected to decline in the future.

2. **C** Rational investors feel that long-term bonds have more risk exposure than short-term securities (i.e., long-term securities are less liquid and subject to more price volatility). The other statements are correct.

3. **C** The market segmentation theory holds that certain types of financial institutions and investors prefer to confine (most of) their investment activity to certain maturity ranges of the fixed-income market and that supply and demand forces within each segment ultimately determine the shape of the yield curve.

4. **C** Open market operations are carried on frequently. The Fed's selling of Treasuries in the open market takes money out of the economy, reducing the amount of loanable funds and increasing interest rates. The opposite occurs when the Fed buys Treasuries in the open market.

5. **C** The yield ratio is 1 + relative yield spread, or 1 + 0.0714 = 1.0714.

6. **D** Absolute yield spread = yield on the 3-year corporate issue – yield on the on-the-run 3-year Treasury issue = 7.00% – 5.00% = 2.00% or 200 bp.

7. **A** The yield on the corporate is 7 percent, so the relative yield is $\dfrac{7\%-6\%}{6\%}$, which is 1/6 or 16.67% of the 3-year Treasury yield.

8. **B.** A contracting economy means lower corporate earnings which increases the probability of default on debt and increases yield spreads between corporate issues and Treasuries at a particular maturity.

9. **D** yield ratio $= \dfrac{\text{higher yield bond}}{\text{lower yield bond}} = 1.0833$. Given the higher yield is 6.5%, the lower yield can be calculated as:

   $$\frac{6.5\%}{1.0833} = \text{lower yield bond} = 6.0\%.$$

10. **A** The tax rate that makes investors indifferent between two otherwise equivalent bonds is determined by solving for the tax rate in the equation: tax-exempt yield = (1 – tax rate) × taxable yield. Rearranging this relationship, we have:

$$\text{marginal tax rate} = 1 - \frac{\text{tax-exempt rate}}{\text{taxable rate}} = 1 - \frac{6.2}{8.2} = 24.39\%.$$

11. **B** As an economy expands, credit spreads decline as expected corporate earnings rise. This is because, with stronger earnings, corporations are less likely to default on their debt.

12. **C** The less liquidity a bond has, the higher its yield spread relative to Treasuries. This is because investors require a higher yield to compensate them for giving up liquidity, which results in a greater spread over Treasury issues, which are very liquid.

13. **B** Since we usually speak of credit spreads as yield spreads to Treasuries, a change in the yield spread does not imply any change in the values of Treasuries.

14. **C** The taxable-equivalent yield on this municipal bond is $\frac{5.25}{(1-0.31)} = \frac{5.25}{0.69} = 7.61\%$. Since this is higher than the yield on the (taxable) corporate bond, the municipal bond is preferred. Alternatively, the after-tax yield on the corporate is 7.5% (1 – 0.31) = 5.175%, which is less than the tax-exempt yield, leading to the same decision.

The following is a review of the Debt Investments: Analysis and Valuation principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

INTRODUCTION TO THE VALUATION OF DEBT SECURITIES

Study Session 15

EXAM FOCUS

Bond valuation is all about calculating the present value of the promised cash flows. If your time-value-of-money skills are not up to speed, take the time now to revisit the Study Session 2 review of TVM concepts. The material in this topic review is very important. Calculating the value of a bond by discounting expected cash flows should become an easy exercise. The final material on discounting a bond's expected cash flows using spot rates and the idea of an "arbitrage-free" bond valuation is quite important as well. A good understanding here will just make what follows all the easier to understand.

LOS 66.a: Describe the fundamental principles of bond valuation.

The general procedure for valuing fixed income securities (or any security) is to take the present values of all the expected cash flows and add them up to get the value of the security.

There are three steps in the bond valuation process:

1. **Estimate the cash flows** over the life of the security. For a bond, there are two types of cash flows: (1) the coupon payments and (2) the return of principal.

2. **Determine the appropriate discount rate** based on the risk of (uncertainty about) the receipt of the estimated cash flows.

3. **Calculate the present value of the estimated cash flows** by multiplying the bond's expected cash flows by the appropriate discount factors.

LOS 66.b: Identify the types of bonds for which estimating the expected cash flows is difficult, and explain the problems encountered when estimating the cash flows for these bonds.

Certainly, one problem in estimating future cash flows for bonds is predicting defaults and any potential credit problems that make the receipt of future cash flows uncertain. Aside from credit risk, however, we can identify three situations where estimating future cash flows poses additional difficulties.

1. **The principal repayment stream is not known with certainty.** This category includes bonds with embedded options (puts, calls, prepayment options, and accelerated sinking fund provisions). For these bonds, the future stream of principal payments is uncertain and will depend to a large extent on the future path of interest rates. For example, lower rates will increase prepayments of mortgage passthrough securities and principal will be repaid earlier.

2. **The coupon payments are not known with certainty.** With floating-rate securities, future coupon payments depend on the path of interest rates. With some floating-rate securities, the coupon payments may depend on the price of a commodity or the rate of inflation over some future period.

3. **The bond is convertible or exchangeable into another security.** Without information about future stock prices and interest rates, we don't know when the cash flows will come or how large they will be.

LOS 66.c: Determine the appropriate interest rates for valuing a bond's cash flows, compute the value of a bond, given the expected annual or semiannual cash flows and the appropriate single (constant) or multiple (arbitrage-free rate curve) discount rates, explain how the value of a bond changes if the discount rate increases or decreases, and compute the change in value that is attributable to the rate change, and explain how the price of a bond changes as the bond approaches its maturity date, and compute the change in value that is attributable to the passage of time.

For a Treasury bond, the appropriate rate used to value the promised cash flows is the risk-free rate. This may be a single rate, used to discount all of the cash flows, or a series of discount rates that correspond to the times until each cash flow arrives.

For non-Treasury securities, we must add a risk premium to the risk-free (Treasury) rate to determine the appropriate discount rate. This risk premium is one of the yield spread measures covered in the previous review and is the added yield to compensate for greater credit risk, liquidity risk, call risk, prepayment risk, etc. When using a single discount rate to value bonds, the risk premium is added to the risk-free rate to get the appropriate discount rate for all of the expected cash flows.

yield on a risky bond = yield on a default-free bond + risk premium

Other things being equal, the riskier the security, the higher the yield differential (or risk premium) we need to add to the on-the-run Treasury yields.

Computing the Value of a Bond

Valuation with a single yield (discount rate). Recall that we valued an annuity using the time value of money keys on the calculator. For an option-free coupon bond, the coupon payments can be valued as an annuity. In order to take into account the payment of the par value at maturity, we will enter this final payment as the future value. This is the basic difference between valuing a coupon bond and valuing an annuity.

For simplicity, consider a security that will pay \$100 per year for ten years and make a single \$1,000 payment at maturity (in ten years). If the appropriate discount rate is 8 percent for all the cash flows, the value is:

$$\frac{100}{1.08}+\frac{100}{1.08^2}+\frac{100}{1.08^3}+\frac{100}{1.08^4}+\ldots\ldots+\frac{100}{1.08^{10}}+\frac{1{,}000}{1.08^{10}}=\$1{,}134.20=\text{present value of expected cash flows}$$

This is simply the sum of the present values of the future cash flows, \$100 per year for 10 years and \$1,000 (the principal repayment) to be received at the end of the tenth year, at the same time as the final coupon payment.

The calculator solution is:

N = 10; PMT = 100; FV = 1,000; I/Y = 8; CPT → PV = –\$1,134.20

N = number of years; PMT = the *annual* coupon payment; I/Y = the *annual* discount rate; and FV = the par value or selling price at the end of an assumed holding period.

Professor's Note: Take note of a couple of points here. The discount rate is entered as a whole number in percent, 8, not 0.08. The ten coupon payments of \$100 each are taken care of in the N=10 entry, the principal repayment is in the FV=1,000 entry. Lastly, note that the PV is negative; it will be the opposite sign to the sign of PMT and FV. The calculator is just "thinking" that if you receive the payments and future value (you own the bond) then you must pay the present value of the bond today (you must buy the bond). That's why the PV amount is negative; it is a cash outflow to a bond buyer. Just make sure that you give the payments and future value the same sign, and then you can ignore the sign on the answer (PV).

Valuation using multiple rates. Rather than use a single discount rate for all the cash flows, you could be given information on the appropriate rate to use for each individual cash flow, based on how far in the future it will be received. The rates that are specific to the time periods until each cash flow is to be received are called *spot rates.* The spot-rate yield curve that can be used to correctly price on-the-run Treasury securities is called an arbitrage-free rate curve. We will learn more about spot rates and the concept of an arbitrage-free rate curve later in this study session.

Consider a security that will make three annual payments of $100 and return a $1,000 principal value at the end of three years (at maturity). You are given the following (annual) spot rates:

1 year = 7%, 2 years = 8%, 3 years = 9%

In this case we calculate the sum of the present values of the cash flows, using the discount rates that correspond to the time until each cash flow is received, as:

$$\frac{100}{1.07}+\frac{100}{1.08^2}+\frac{100}{1.09^3}+\frac{1{,}000}{1.09^3}=\$1{,}028.59=\text{present value of expected cash flows}$$

Here, we have illustrated the valuation of a bond that makes annual payments based on a series of annual spot rates. Since U.S. bonds typically make semiannual coupon payments, the convention is to quote yields as twice the semiannual discount rate. This particular yield convention is called the *semiannual-pay yield to maturity or bond equivalent yield* and is formally introduced and defined in the next topic review. Remember that in general, when valuing bonds or calculating the present value of a regular annuity, we must match the period of the discount rate to the frequency of the payments. If we have semiannual payments, calculate the present value using semiannual discount rates and the number of *semiannual* periods. An example will illustrate this method.

Example: Bond valuation with semiannual coupons

A 5-year Treasury note has a 9 percent coupon rate and a (semiannual) yield to maturity of 10 percent. What is its market value?

Answer:

Each $1,000 face value bond will make ten semiannual payments of $45 = (0.09/2) × 1,000, and a payment of $1,000 with the last coupon interest payment at the end of ten semiannual periods. The appropriate discount rate for the semiannual payments is one-half the yield to maturity or 5 percent.

N = 10; PMT = 45; FV = 1,000; I/Y = 5; CPT→PV = $961.39 per $1,000 par value or 96.139% of par value

(*See Exam Flashback #1.*)

The Change in Value When Interest Rates Change

Bond values and bond yields are inversely related. An *increase* in the discount rate will *decrease* the present value of a bond's expected cash flows; a decrease in the discount rate will increase the present value of a bond's expected cash flows. The change in bond value in response to a change in the discount rate can be calculated as the difference between the present values of the cash flows at the two different discount rates.

Example: Changes in required yield

A bond has a par value of $1,000, a 6 percent semiannual coupon, and three years to maturity. **Compute** the bond values when the yield to maturity is 3, 6, and 12 percent.

Answer:

$$\text{At I/Y} = \frac{3}{2};\ \text{N} = 3 \times 2;\ \text{FV} = 1{,}000;\ \text{PMT} = \frac{60}{2};\ \text{CPT} \rightarrow \text{PV} = -1{,}085.458$$

$$\text{At I/Y} = \frac{6}{2};\ \text{N} = 3 \times 2;\ \text{FV} = 1{,}000;\ \text{PMT} = \frac{60}{2};\ \text{CPT} \rightarrow \text{PV} = -1{,}000.000$$

$$\text{At I/Y} = \frac{12}{2};\ \text{N} = 3 \times 2;\ \text{FV} = 1{,}000;\ \text{PMT} = \frac{60}{2};\ \text{CPT} \rightarrow \text{PV} = -852.480$$

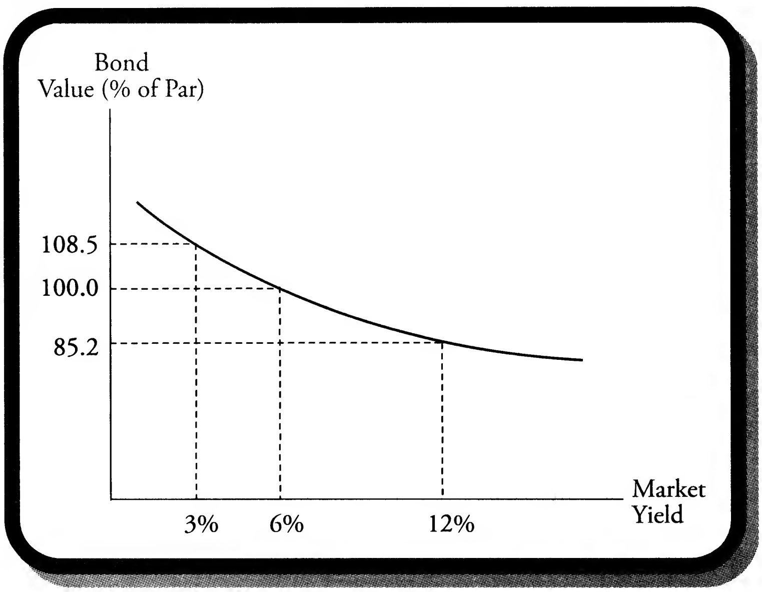
We have illustrated here a point covered earlier; if the yield to maturity equals the coupon rate, the bond value is equal to par. If the **yield to maturity is higher (lower) than the coupon rate, then the bond is trading at a discount (premium) to par.**

We can now calculate the percentage change in price for changes in yield. If the required yield decreases from 6% to 3%, the value of the bond increases by $\frac{1{,}085.46}{1{,}000.00} - 1 = 8.546\%$. If the yield increases from 6% to 12%, the bond value decreases by $\frac{852.48}{1{,}000.00} - 1 = -14.752\%$.

Professor's Note: Notice that in these calculations, you only need to change the interest rate (I/Y) and then compute PV once the values of N, PMT, and FV have been entered. The TVM keys "remember" the values for these inputs, even after the calculator has been turned off!

Price-yield profile. If you plot a bond's yield to its corresponding value, you'll get a graph like the one shown in Figure 1. Here we see that higher prices are associated with lower yields. This graph is called the *price-yield curve*. Note that it is not a straight line but is curved. For option-free bonds the price-yield curve is convex toward the origin, meaning it looks like half of a smile.

Figure 1: The Price-Yield Profile



The Price of a Bond as the Bond Approaches Its Maturity Date

Prior to maturity, a bond can be selling at a significant discount or premium to par value. However, regardless of its required yield, the price will converge to par value as maturity approaches. Consider the bond in the previous example ($1,000 par value, 3-year life, paying 6 percent semiannual coupons). The bond values corresponding to required yields of 3, 6, and 12 percent as the bond approaches maturity are presented in Figure 2.

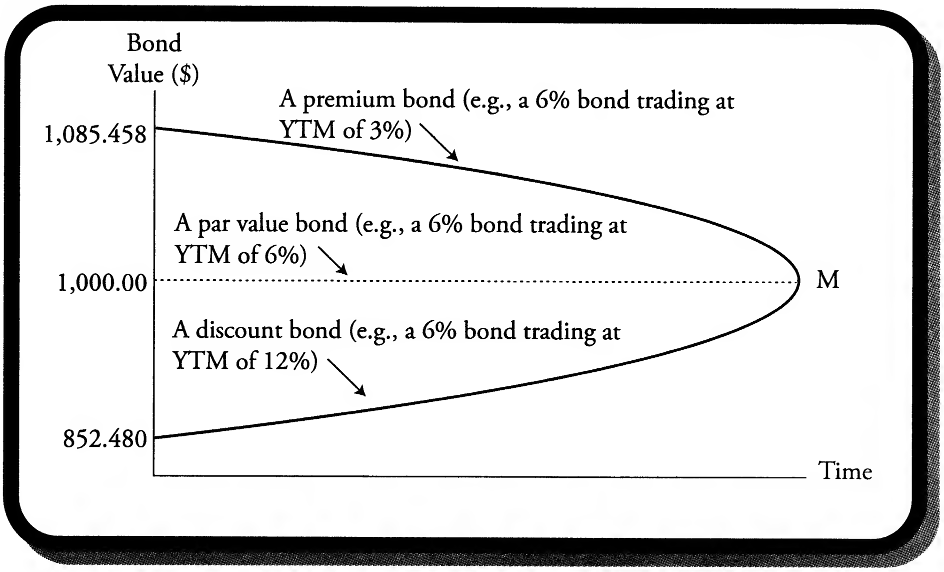
Figure 2: Bond Values and the Passage of Time

Time to Maturity	*YTM = 3%*	*YTM = 6%*	*YTM = 12%*
3.0 years	$1,085.4	$1,000.00	$852.48
2.5	1,071.74	1,000.00	873.63
2.0	1,057.82	1,000.00	896.05
1.5	1,043.68	1,000.00	919.81
1.0	1,029.34	1,000.00	945.00
0.5	1,014.78	1,000.00	971.69
0.0	1,000.00	1,000.00	1,000.00

To compute the change in bond value due to the passage of time, just revalue the bond with the number of periods (remaining until maturity) reduced. Note that in the above example, the value of a 6 percent bond with three years until maturity and a yield to maturity of 3 percent is: FV = 1,000; PMT = 30; N = 6; I/Y = 1.5; CPT → PV = $1,085.46. To see the effect of the passage of time (with the yield to maturity held constant) just enter: N = 5 CPT → PV to get the value one period (six months) from now of $1,071.74, or N = 4 CPT → PV to get the value two periods (one year) from now of $1,057.82.

The change in value associated with the passage of time for the three bonds represented in Figure 2 is presented graphically in Figure 3.

Figure 3: Premium, Par, and Discount Bonds



LOS 66.d: Compute the value of a zero-coupon bond, explain the arbitrage-free valuation approach and the market process that forces the price of a bond toward its arbitrage-free value, determine whether a bond is undervalued or overvalued, given the bond's cash flows, appropriate spot rates or yield to maturity, and current market price, explain how a dealer could generate an arbitrage profit.

Since a zero-coupon bond has only a single payment at maturity, the value of a zero is simply the present value of the par or face value. Given the yield to maturity, the calculation is:

$$\text{bond value} = \frac{\text{maturity value}}{(1+i)^{\text{number of years} \times 2}}$$

Note that this valuation model requires just three pieces of information:

- The bond's maturity value, assumed to be \$1,000.
- The semiannual discount rate, *i*.
- The life of the bond, *N* years.

Alternatively, using the TVM keys, we enter:

PMT = 0; FV = par; N = # years × 2; I/Y = YTM/2 = semiannual discount rate; CPT → PV

Although zero-coupon bonds do not pay coupons, it is customary to value zero-coupon bonds using semiannual discount rates. Note that *N* is now two times the number of years to maturity and the semiannual discount rate is one-half the yield to maturity expressed as a BEY.

Example: Valuing a zero-coupon bond

Compute the value of a 10-year, \$1,000 face value zero-coupon bond with a yield to maturity of 8 percent.

Answer:

To find the value of this bond given its yield to maturity of 8 percent (a 4 percent semiannual rate), we can calculate:

$$\text{bond value} = \frac{1{,}000}{\left(1+0.08/2\right)^{10\times2}} = \frac{1{,}000}{(1.04)^{20}} = \$456.39$$

Or, using a calculator, use the following inputs:

$$N = 10 \times 2 = 20;\ FV = 1{,}000;\ I/Y = 8/2 = 4;\ PMT = 0;\ CPT \rightarrow PV = -\$456.39$$

The difference between the current price of the bond (\$456.39) and its par value (\$1,000) is the amount of compound interest that will be earned over the 10-year life of the issue.

Professor's Note: Exam questions will likely specify whether annual or semiannual discounting should be used. Just be prepared to value a zero-coupon bond either way.

Arbitrage-Free Bond Valuation

Yield to maturity is a summary measure and is essentially an internal rate of return based on a bond's cash flows and its market price. In the traditional valuation approach, we get the yield to maturity of bonds with maturity and risk characteristics similar to those of the bond we wish to value. Then we use this rate to discount the cash flows of the bond to be valued.

With the **arbitrage-free valuation approach**, we *discount each cash flow using a discount rate that is specific to the maturity of each cash flow*. Again, these discount rates are called **spot rates**, and can be thought of as the required rates of return on zero-coupon bonds maturing at various times in the future.

The arbitrage-free valuation approach simply says that the value of a Treasury bond based on (Treasury) spot rates must be equal to the value of the "parts" (i.e., the sum of the present values of all of the expected cash flows). If this is not the case, there must be an arbitrage opportunity. If a bond is selling for less than the sum of the present values of its expected cash flows, an arbitrageur will buy the bond and sell the "pieces." If the bond is selling for more than the sum of the values of the pieces (individual cash flows), one could buy the pieces, package them to "make" a bond, and then sell the bond "package" to earn an arbitrage profit.

The first step in checking for arbitrage-free valuation is to value a coupon bond using the appropriate spot rates. The second step is to compare this value to the market price of the bond. If the computed value is not equal to the market price, there is an arbitrage profit to be earned by buying the lower-priced alternative (either the bond or the individual cash flows) and selling the higher-priced alternative. Of course, this assumes that there are zero-coupon bonds available that correspond to the coupon bond's cash flows.

Example: Arbitrage-free valuation

Consider a 6 percent Treasury note with 1.5 years to maturity. Spot rates (expressed as semiannual yields to maturity) are: 6 months = 5%, 1 year = 6%, 1.5 years = 7%. If the note is selling for $992, **compute** the arbitrage profit and **explain** how a dealer would perform the arbitrage.

Answer:

To value the note, note that the cash flows (per $1,000 par value) will be $30, $30, and $1,030, and that the semiannual discount rates are one-half the stated yield to maturity.

Using the semiannual spot rates, the present value of the expected cash flows is:

$$\text{present value using spot rates} = \frac{30}{1.025} + \frac{30}{1.03^2} + \frac{1,030}{1.035^3} = \$986.55$$

This value is less than the market price of the note, so we will buy the individual cash flows (zero-coupon bonds), combine them into a 1.5-year note "package" and sell the package for the market price of the note. This will result in an immediate and riskless profit of 992.00 – 986.55 = $5.45 per bond.

Determining whether a bond is over- or undervalued is a two-step process. First, compute the value of the bond using either the spot rates or yield to maturity, remembering that both are often given as two times the semiannual discount rate(s). Second, compare this value to the market price given in the problem to see which is higher.

How a Dealer Can Generate an Arbitrage Profit

Recall that the Treasury STRIPS program allows dealers to divide Treasury bonds into their coupon payments (by date) and their maturity payments, in order to create zero-coupon securities. The program also allows reconstitution of Treasury bonds/notes by putting the individual cash flows back together to create Treasury securities. Ignoring any costs of performing these transformations, the ability to separate and reconstitute Treasury securities will insure that the arbitrage-free valuation condition is met.

The STRIPS program allows for just the arbitrage we outlined above. If the price of the bond is greater than its arbitrage-free value, a dealer could buy the individual cash flows and sell the package for the market price of the bond as we detailed previously. If the price of the bond is less than its arbitrage-free value, an arbitrageur can make an immediate and riskless profit by purchasing the bond and selling the parts for more than the cost of the bond.

Such arbitrage opportunities and the related buying of bonds priced "too low" and sales of bonds priced "too high" will force the bond prices toward equality with their arbitrage-free values, eliminating further arbitrage opportunities.

Key Concepts

1. To value a bond, one must estimate the amount and timing of the cash flows from coupons and par value, determine the appropriate discount rate(s), and calculate the sum of the present values of the cash flows.
2. Certain bond features, including embedded options, convertibility, or floating rates, can make the estimation of future cash flows uncertain, which adds complexity to the estimation of bond values.
3. In bond valuation, the approximate discount rate is a function of a risk-free rate (e.g., U.S. Treasury yields) and a risk premium (the incremental yield spread required to compensate the investor for incurring any additional risks relative to Treasury securities).
4. When discounting a bond's expected cash flows, the period of the discount rate must match the frequency of the payments.
5. The required yield on a bond can change during the life of the bond, but the cash flows generally will not. Therefore, decreases (increases) in required yield will increase (decrease) the bond's price.
6. When interest rates (yields) do not change, a bond's price will move toward its par value as time passes and the maturity date approaches.
7. The value of a zero-coupon bond calculated using a semiannual discount rate (i) is:

   $$\text{bond value} = \frac{\text{maturity value}}{(1+i)^{\text{number of years} \times 2}}$$

8. If the required yield equals the coupon rate, the bond will trade at par. If the required yield exceeds the coupon rate, the bond will trade at a discount. If the required yield is less than the coupon rate, the bond will trade at a premium.
9. YTM is really an average of the required rates for the individual cash flows of a bond; the required rate for an individual cash flow is called the spot rate.
10. Bond prices are considered to be arbitrage-free if the sum of the present values of the individual cash flows, discounted using spot rates, is equal to the bond price.
11. If bonds prices are not equal to their arbitrage-free values, dealers can generate arbitrage profits by buying the lower-priced alternative (either the bond or the individual cash flows) and selling the higher-priced alternative (either the individual cash flows or a package of the individual cash flows equivalent to the bond).

Exam Flashbacks

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #102 from '00–'03 sample exams.

A newly issued 10-year option-free bond is valued at par on June 1, 2000. The bond has an annual-pay coupon of 8.0 percent. On June 1, 2003, the bond will have a yield to maturity of 7.1 percent. The first coupon is reinvested at 8.0 percent and the second coupon is reinvested at 7.0 percent. The future price of the bond on June 1, 2003 is *closest* to:
A. 100.0% of par.
B. 102.5% of par.
C. 104.8% of par.
D. 105.4% of par.

Exam Flashback # 2

Source: Question #47 from '91 actual exam.

Using semiannual compounding, a 15-year, zero-coupon bond that has a par value of $1,000 and a required return of 8% would be priced at:
A. $308.
B. $315.
C. $464.
D. $555.

Exam Flashback # 3

Source: Question #108 from '99–'03 sample exams.

If an investor's required return is 12 percent, the value of a 10-year maturity zero-coupon bond with a maturity value of $1,000 is *closest* to:
A. $312.
B. $688.
C. $1,000.
D. $1,312.

Exam Flashback # 4

Source: Question #Source: Question #41 from '90–'91 actual exams

Consider a five-year bond with a 10 percent coupon that has a present yield-to-maturity of 8 percent. If interest rates remain constant, one year from now the price of this bond will be:
A. higher.
B. lower.
C. the same.
D. cannot be determined.

Concept Checkers: Introduction to the Valuation of Debt Securities

1. An analyst observes a 5-year, 10 percent coupon bond with semiannual payments. The face value is £1,000. How much is each coupon payment?
   A. £50.
   B. £25.
   C. £500.
   D. £100.

2. A 20-year, 10 percent annual-pay bond has a par value of $1,000. What would this bond be trading for if it were being priced to yield 15 percent as an annual rate?
   A. $685.14.
   B. $687.03.
   C. $828.39.
   D. $832.40.

3. An analyst observes a 5-year, 10 percent semiannual-pay bond. The face amount is £1,000. The analyst believes that the yield to maturity for this bond should be 15 percent. Based on this yield estimate, the price of this bond would be:
   A. £828.40.
   B. £832.39.
   C. £1,189.53.
   D. £1,193.04.

4. Two bonds have par values of $1,000. Bond A is a 5 percent annual-pay, 15-year bond priced to yield 8 percent as an annual rate; the other (Bond B) is a 7.5 percent annual-pay, 20-year bond priced to yield 6 percent as an annual rate. The values of these two bonds would be:

	Bond A	Bond B
A.	$740.61	$847.08
B.	$740.61	$1,172.04
C.	$743.22	$1,172.04
D.	$743.22	$847.08

5. Bond A is a 15-year, 10.5 percent semiannual-pay bond priced with a yield to maturity of 8 percent, while Bond B is a 15-year, 7 percent semiannual-pay bond priced with the same yield to maturity. Given that both bonds have par values of $1,000, the prices of these two bonds would be:

	Bond A	Bond B
A.	$1,216.15	$913.54
B.	$1,216.15	$944.41
C.	$746.61	$944.41
D.	$746.61	$913.54

Use the following data to answer Questions 6 through 8.

An analyst observes a 20-year, 8 percent option-free bond with semiannual coupons. The required semiannual-pay yield to maturity on this bond was 8 percent, but suddenly it drops to 7.25 percent.

6. Thus, the price of this bond:
   A. will increase.
   B. will decrease.
   C. will stay the same.
   D. cannot be determined without additional information.

7. Prior to the change in the required yield, what was the price of the bond?
   A. 92.64.
   B. 100.00.
   C. 107.85
   D. Cannot be determined without additional information.

8. The percentage change in the price of this bond when the rate decreased is *closest* to:
   A. 7.86%.
   B. 7.79%.
   C. 8.00%.
   D. 8.15%.

9. Treasury spot rates (expressed as semiannual-pay yields to maturity) are as follows: 6 months = 4 percent, 1 year = 5 percent, 1.5 years = 6 percent. A 1.5-year, 4 percent Treasury note is trading at $965. The arbitrage trade and arbitrage profit are:
   A. Buy the bond, sell the pieces, earn $7.09 per bond.
   B. Sell the bond, buy the pieces, earn $7.09 per bond.
   C. Buy the bond, sell the pieces, earn $7.91 per bond.
   D. Sell the bond, buy the pieces, earn $7.91 per bond.

10. A $1,000, 5 percent, 20-year annual-pay bond has a yield of 6.5 percent. If the yield remains unchanged, how much will the bond value increase over the next 3 years?
    A. $13.58.
    B. $13.62.
    C. $13.78.
    D. $13.96.

11. The value of a 17-year zero-coupon bond with a maturity value of $100,000 and a semiannual-pay yield of 8.22 percent is *closest* to:
    A. $24,618.
    B. $25,425.
    C. $26,108.
    D. $91,780.

Answers – Exam Flashbacks

1. **C** The question is asking for the value of the bond 3 years from June 1, 2000.

   N = 10 – 3 = 7; I/Y = 7.1; PMT = 80; FV = \$1,000. Compute PV = 1,048 or 104.8% of par. The information regarding reinvestment rates is irrelevant to the question.

2. **A** Semiannual math: N = 15 × 2 = 30; FV = \$1,000; I/Y = 8 / 2; CPT → PV = \$308

   Note: If you had incorrectly assumed annual compounding, you would have computed:
   N = 15; FV = \$1,000; I/Y = 8; CPT → PV = \$315

3. **A** Using a semiannual-compounding assumption, I/Y = 6; N = 20; FV = \$1,000; CPT → PV = \$311.80. Note that here annual discounting gives a value of \$322, and A is still the best answer.

4. **B** A discount bond is sold below par; as time passes its value increases upward to par. A premium bond is sold at a price above par; as time passes its value decreases toward par. The bond given is a premium bond.

Answers – Concept Checkers: Introduction to the Valuation of Debt Securities

1. **A** $\text{CPN} = 1{,}000 \times \frac{0.10}{2} = £50$

2. **B** $\text{bond value} = \sum_{t=1}^{20} \frac{100}{(1+0.15)^t} + \frac{1{,}000}{(1+0.15)^{20}} = \$687.03$

   N = 20; I/Y = 15; FV = 1,000; PMT = 100; CPT → PV = –\$687.03

3. **A** N = 10; I/Y = 7.5; FV = 1,000; PMT = 50; CPT → PV = –\$828.40

4. **C** Bond A: N = 15; I/Y = 8; FV = 1,000; PMT = 50; CPT → PV = –\$743.22

   Bond B: N = 20; I/Y = 6; FV = 1,000; PMT = 75; CPT → PV = –\$1,172.04

   Because the coupon on Bond A is less than its required yield, the bond will sell at a discount; conversely, because the coupon on Bond B is greater than its required yield, the bond will sell at a premium.

5. **A** Bond A: $N = 15 \times 2 = 30$; $I/Y = \frac{8}{2} = 4$; FV = 1,000; $PMT = \frac{105}{2} = 52.50$; CPT → PV = –\$1,216.15

   Bond B: $N = 15 \times 2 = 30$; $I/Y = \frac{8}{2} = 4$; FV = 1,000; $PMT = \frac{70}{2} = 35$; CPT → PV = –\$913.54

6. **A** The price-yield relationship is inverse. If the required yield falls, the bond's price will rise, and vice versa.

7. **B** If YTM = stated coupon rate ⇒ bond price = 100 or par value.

8. **A** The new value is $40 = \text{N}, \ \frac{7.25}{2} = \text{I/Y}, \ 40 = \text{PMT}, \ 1{,}000 = \text{FV}$
   $\text{CPT} \rightarrow \text{PV} = -1{,}078.55$, an increase of 7.855%

9. **A** $\text{Arbitrage-free value} = \frac{20}{1.02} + \frac{20}{1.025^2} + \frac{1020}{1.03^3} = \$972.09$

   Since the bond price (\$965) is less, buy the bond and sell the pieces for an arbitrage profit of \$7.09 per bond.

10. **B** With 20 years to maturity, the value of the bond with an annual-pay yield of 6.5% is: 20 = N, 50 = PMT, 1,000 = FV, 6.5 = I/Y, CPT – PV = –834.72. With 17 = N, CPT → PV = –848.34, so the value will increase $13.62.

11. **B** $\text{PMT} = 0,\ \text{N} = 2 \times 17 = 34,\ \text{I/Y} = \frac{8.22}{2} = 4.11,\ \text{FV} = 100{,}000$

$\text{CPT} \rightarrow \text{PV} = -25{,}424.75$, or

$$\frac{100{,}000}{(1.0411)^{34}} = \$25{,}424.76$$

The following is a review of the Debt Investments: Analysis and Valuation principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

YIELD MEASURES, SPOT RATES, AND FORWARD RATES

Study Session 15

EXAM FOCUS

This topic review gets a little more specific about yield measures and introduces some yield measures that you will (almost certainly) need to know for the exam: current yield, yield to maturity, and yield to call. Please pay particular attention to the concept of a bond equivalent yield and how to convert various yields to a bond equivalent basis. The other important thing about the yield measures here is to understand what they are telling you so that you understand their limitations. The Level 1 exam may place as much emphasis on these issues as on actual yield calculations.

The final section of this review introduces forward rates. The relationship between forward rates and spot rates is an important one. At a minimum, you should be prepared to solve for spot rates given forward rates and to solve for an unknown forward rate given two spot rates. You should also get a firm grip on the concept of an option-adjusted spread, when it is used and how to interpret it, as well as how and why it differs from a zero-volatility spread.

LOS 67.a: Explain the sources of return from investing in a bond (i.e., coupon interest payments, capital gain/loss, reinvestment income).

Debt securities that make explicit interest payments have **three sources of return:**

1. The periodic *coupon interest payments* made by the issuer.

2. The *recovery of principal, along with any capital gain or loss* that occurs when the bond matures, is called, or is sold.

3. *Reinvestment income*, or the income earned from reinvesting the periodic coupon payments (i.e., the compound interest on reinvested coupon payments).

The interest earned on reinvested income is an important source of return to bond investors. The uncertainty about how much reinvestment income a bondholder will realize is what we have previously addressed as *reinvestment risk.*

LOS 67.b: Compute the traditional yield measures for fixed-rate bonds (e.g., current yield, yield to maturity, yield to first call, yield to first par call date, yield to refunding, yield to put, yield to worst, cash flow yield) and explain the assumptions underlying traditional yield measures and the limitations of the traditional yield measures.

Current yield is the simplest of all return measures, but it offers limited information. This measure looks at just one source of return: *a bond's annual interest income*—it does not consider capital gains/losses or reinvestment income. The formula for the current yield is:

$$\text{current yield} = \frac{\text{annual cash coupon payment}}{\text{bond price}}$$

Example: Computing current yield

Consider a 20-year, $1,000 par value, 6 percent *semiannual-pay* bond that is currently trading at $802.07. **Calculate** the current yield.

Answer:

The *annual* cash coupon payments total:

annual cash coupon payment = par value × stated coupon rate = $1,000 × 0.06 = $60

Since the bond is trading at $802.07, the current yield is:

$$\text{current yield} = \frac{60}{802.07} = 0.0748\text{, or } 7.48\%$$

Note that current yield is based on *annual* coupon interest so that it is the same for a semiannual-pay and annual-pay bond with the same coupon rate and price. (*See Exam Flashback #1.*)

Yield to maturity (YTM) is an annualized internal rate of return, based on a bond's price and its promised cash flows. For a bond with semiannual coupon payments, the yield to maturity is stated as two times the semiannual internal rate of return implied by the bond's price. The formula that relates bond price and YTM for a semiannual coupon bond is:

$$\text{bond price} = \frac{CPN_1}{\left(1 + \frac{YTM}{2}\right)} + \frac{CPN_2}{\left(1 + \frac{YTM}{2}\right)^2} + \cdots + \frac{CPN_{2N} + \text{Par}}{\left(1 + \frac{YTM}{2}\right)^{2N}}$$

where:
CPN_t = the (semiannual) coupon payment received after t semiannual periods
N = number of years to maturity
YTM = yield to maturity

YTM and price contain the same information. That is, given the YTM, you can calculate the price and given the price, you can calculate the YTM.

We cannot easily solve for YTM from the bond price. Given a bond price and the coupon payment amount, we could solve it by trial and error, trying different values of YTM until the present value of the expected cash flows is equal to price. Fortunately, your calculator will do exactly the same thing, only faster. It uses a trial and error algorithm to find the discount rate that makes the two sides of the pricing formula equal.

Example: Computing YTM

Consider a 20-year, $1,000 par value bond, with a 6 percent coupon rate (semiannual payments) that is currently trading at $802.07. **Calculate** the YTM.

Answer:

Using a financial calculator, you'd find the YTM on this bond as follows:

PV = –802.07; N = 20 × 2 = 40; FV = 1,000; PMT = 60/2 = 30; CPT → I/Y = 4.00

4 percent is the semiannual discount rate, $\frac{YTM}{2}$ in the formula, so the YTM = 2 × 4% = 8%.

Note that the signs of PMT and FV are positive and the sign of PV is negative; you must do this to avoid the dreaded "Error 5" message on the TI calculator. If you get the "Error 5" message, you can assume you have not assigned a negative value to the price (PV) of the bond and a positive sign to the cash flows to be received from the bond.

There are certain relationships that exist between different yield measures, depending on whether a bond is trading at par, at a discount, or at a premium. These relationships are shown in Figure 1.

Figure 1: Par, Discount, and Premium Bonds

Bond Selling at:	*Relationship*
Par	coupon rate = current yield = yield to maturity
Discount	coupon rate < current yield < yield to maturity
Premium	coupon rate > current yield > yield to maturity

These conditions will hold in all cases; every discount bond will have a nominal yield (coupon rate) that is less than its current yield and a current yield that is less than its YTM.

The yield to maturity calculated in the previous example (2 × the semiannual discount rate) is referred to as the **bond equivalent yield (BEY)**. If you are given yields that are identified as BEY, you will know that you must divide by two to get the semiannual discount rate. With those bonds that make annual coupon payments we can calculate an **annual yield to maturity**, which is simply the internal rate of return for the expected annual cash flows.

Example: Calculating YTM for annual coupon bonds

Consider an annual-pay 20-year, $1,000 par value, with a 6 percent coupon rate that is trading at $802.07. **Calculate** the *annual-pay YTM.*

Answer:

The relation of price and annual-pay YTM on this bond is:

$$802.07 = \sum_{t=1}^{20} \frac{60}{(1+\text{YTM})^t} + \frac{1{,}000}{(1+\text{YTM})^{20}} \Rightarrow \text{YTM} = 8.019\%$$

Here we have separated the coupon cash flows and the principal repayment.

The calculator solution is:

PV = –802.07; N = 20; FV = 1,000; PMT = 60; CPT → I/Y = 8.019; 8.019% is the annual-pay YTM.

Use a discount rate of 8.019 percent, and you'll find the present value of the bond's future cash flows (annual coupon payments and the recovery of principal) will equal the current market price of the bond. *The discount rate is the bond's YTM.*

For zero-coupon Treasury bonds, the convention is to quote the yields as BEYs (semiannual-pay YTMs).

Example: Calculating YTM for zero-coupon bonds

A 5-year Treasury STRIP is priced at $768. **Calculate** the semiannual-pay YTM and annual-pay YTM.

Answer:

The direct calculation method, based on the geometric mean covered in Quantitative Methods, is:

$$\text{the semiannual-pay YTM or BEY} = \left[\left(\frac{1{,}000}{768}\right)^{\frac{1}{10}} - 1\right] \times 2 = 5.35\%$$

$$\text{the annual-pay YTM} = \left(\frac{1{,}000}{768}\right)^{\frac{1}{5}} - 1 = 5.42\%$$

Using the TVM calculator functions:

PV = –768; FV =1,000; PMT = 0; N = 10; CPT→I/Y = 2.675% × 2 = 5.35% for the semiannual-pay YTM, and PV = –768; FV =1,000; PMT = 0; N = 5; CPT→I/Y = 5.42% for the annual-pay YTM.

The annual-pay YTM of 5.42% means that $768 earning compound interest of 5.42%/yr. would grow to $1,000 in five years.

(*See Exam Flashbacks #2 through #5*)

The **yield to call** is used to calculate the yield on callable bonds that are selling at a premium to par. For bonds trading at a premium to par, the *yield to call* may be less than the yield to maturity. This can be the case when the call price is below the current market price.

The calculation of the yield to call is the same as the calculation of yield to maturity, except that the *call price is substituted* for the par value in FV and the *number of semiannual periods until the call date is substituted* for periods to maturity, N. When a bond has a period of call protection, we calculate the **yield to first call** over the period until the bond may first be called, and use the first call price in the calculation as FV. In a similar manner, we can calculate the yield to any subsequent call date using the appropriate call price.

If the bond contains a provision for a call at *par* at some time in the future, we can calculate the *yield to first par call* using the number of years until the par call date and par for the maturity payment. If you have a good understanding of the yield to maturity measure, the YTC is not a difficult calculation; just be very careful about the number of years to the call and the call price for that date. An example will illustrate the calculation of these yield measures.

Example: Computing the YTM, YTC, and yield to first par call

Consider a 20-year, 10% semiannual-pay bond priced at 112 that can be called in five years at 102 and called at par in seven years. **Calculate** the YTM, YTC, and yield to first par call.

Professor's Note: Bond prices are often expressed as a percent of par (e.g., 100 = par).

Answer:

The **YTM** can be calculated as:
N = 40; PV = –112; PMT = 5; FV = 100; CPT→I/Y = 4.361% × 2 = 8.72% = YTM.

To compute the **yield to first call** (YTFC), we substitute the number of semiannual periods until the first call date (10) for N, and the first call price (102) for FV, as follows:

N = 10; PV = –112; PMT = 5; **FV = 102**;

CPT→I/Y = 3.71% and 2 × 3.71 = 7.42% = YTFC

To calculate the **yield to first par call** (YTFPC) we will substitute the number of semiannual periods until the first par call date (14) for N and par (100) for FV as follows:

N = 14; PV = –112; PMT = 5; **FV = 100**; CPT→I/Y = 3.873% × 2 = 7.746% = YTFPC

Note that the yield to call, 7.42 percent, is significantly lower than the yield to maturity, 8.72 percent. If the bond were trading at a discount to par value, there would be no reason to calculate the yield to call. For a discount bond, the YTC will be higher than the YTM since the bond will appreciate more rapidly with the call to at least par and, perhaps, an even greater call price. Bond yields are quoted on a yield to call basis when the YTC is less than the YTM, which can only be the case for bonds trading at a premium to the call price. *(See Exam Flashback #6.)*

The **yield to worst** is the worst yield outcome of any that are possible given the call provisions of the bond. In the above example the yield to first call is less than the YTM and less than the yield to first par call. So the worst possible outcome is a yield of 7.42 percent; the yield to first call is the *yield to worst.*

The **yield to refunding** refers to a specific situation where a bond is currently callable and current rates make calling the issue attractive to the issuer, but where the bond covenants contain provisions giving protection from refunding until some future date. The calculation of the yield to refunding is just like that of YTM or YTC. The difference here is that the yield to refunding would use the call price, but the date (and therefore the number of periods used in the calculation) is the date when refunding protection ends. Recall that bonds that are callable but not currently refundable can be called using funds from sources other than the issuance of a lower coupon bond.

The **yield to put** is used if a bond has a put feature and is selling at a discount. The yield to put will likely be higher than the yield to maturity. The yield to put calculation is just like the yield to maturity with the number of semiannual periods until the put date as N, and the put price as FV.

Example: Computing YTM and yield to put (YTP)

Consider a 3-year, 6 percent, $1,000 *semiannual-pay* bond. The bond is selling for $925.40. The first put opportunity is at par in two years. **Calculate** the YTM and the YTP.

Answer:

Yield to maturity is calculated as:

N = 6; FV = 1,000; PMT = 30; PV = –925.40; CPT→I/Y = 4.44 × 2 = 8.88% = YTM

Yield to put is calculated as:

N = 4; FV = 1,000; PMT = 30; PV = –925.40; CPT→I/Y = 5.11 × 2 = 10.22% = YTP

In this example, the yield to put is higher than the YTM and, therefore, would be the appropriate yield to look at for this bond.

The **cash flow yield** (CFY) is used for mortgage-backed securities and other amortizing asset-backed securities that have monthly cash flows. In many cases, the amount of the principal repayment can be greater than the amount required to amortize the loan over its original life. Cash flow yield (CFY) incorporates an assumed schedule of monthly cash flows based on assumptions as to how prepayments are likely to occur. Once we have projected the monthly cash flows, we can calculate CFY as a *monthly* internal rate of return based on the market price of the security.

Professor's Note: It is unlikely that you will be required to actually calculate a CFY on the exam and more likely that you could be required to interpret one. If you need to calculate a CFY, just use the cash flow keys, put the price of the security as a negative value as CF_0, enter the monthly cash flows sequentially as CFn's, and solve for IRR, which will be a monthly rate.

The following formula is used to convert a (monthly) CFY into bond equivalent form:

$$\text{bond equivalent yield} = \left[(1 + \text{monthly CFY})^6 - 1\right] \times 2$$

Here, we have converted the monthly yield into a semiannual yield and then doubled it to make it equivalent to a semiannual-pay YTM or bond equivalent yield.

A limitation of the CFY measure is that actual prepayment rates may differ from those assumed in the calculation of CFY.

The Assumptions and Limitations of Traditional Yield Measures

The primary **limitation of the yield to maturity measure** is that it does not tell us the compound rate of return that we will realize on a fixed-income investment over its life. This is because we do not know the rate of interest we will realize on the reinvested coupon payments (the reinvestment rate). Reinvestment income can be a significant part of the overall return on a bond. As noted earlier, the uncertainty about the return on reinvested cash flows is referred to as *reinvestment risk.* It is higher for bonds with higher coupon rates, other things equal, and potentially higher for callable bonds as well.

The realized yield on a bond is the actual compound return that was earned on the initial investment. It is usually computed at the end of the investment horizon. For a bond to have a *realized yield* equal to its YTM, all cash flows prior to maturity must be reinvested at the YTM, and the bond must be held until maturity. If the "average" reinvestment rate is below the YTM, the realized yield will be below the YTM. For this reason it is often stated that: *The yield to maturity assumes cash flows will be reinvested at the YTM and assumes that the bond will be held until maturity.* This is the point of LOS here.

The other internal rate of return measures, YTC and YTP, suffer from the same shortcomings since they are calculated like YTMs and do not account for reinvestment income. The CFY measure is also an internal rate of return measure and can differ greatly from the realized yield if reinvestment rates are low, since scheduled principal payments and prepayments must be reinvested along with the interest payments.

LOS 67.c: Explain the importance of reinvestment income in generating the yield computed at the time of purchase, and calculate the amount of income required to generate that yield and discuss the factors that affect reinvestment risk.

Reinvestment income is important because if the reinvestment rate is less than the YTM, the realized yield on the bond will be less than the YTM. The realized yield will always be between the YTM and the assumed reinvestment rate.

If a bondholder holds a bond until maturity and reinvests all coupon interest payments, the total amount generated by the bond over its life has three components:

- Bond principal.
- Coupon interest.
- Interest on reinvested coupons.

Once we calculate the total amount needed for a particular level of compound return over a bond's life, we can subtract the principal and coupon payments to determine the amount of reinvestment income necessary to achieve the target yield. An example will illustrate this calculation.

Example: Calculating required reinvestment income for a bond

If you purchase a 6%, 10-year Treasury bond at par, how much reinvestment income must be generated over its life to provide the investor with a compound return of 6 percent on a semiannual basis?

Answer:

Assuming the bond has a par value of $100, we first calculate the total value that must be generated 10 years (20 semiannual periods) from now as:

$$100(1.03)^{20} = \$180.61$$

There are 20 bond coupons of $3 each, totaling $60, and a payment of $100 of principal at maturity.

Therefore, the required reinvestment income over the life of the bond is:

$$180.61 - 100 - 60 = \$20.61$$

Professor's Note: If we had purchased the bond at a premium or discount, we would still use the purchase price (which would not equal 100) and the required compound return to calculate the total future dollars required, and then subtract the maturity value and the total coupon payments to get the required reinvestment income.

Factors That Affect Reinvestment Risk

Other things being equal, a coupon bond's **reinvestment risk** will *increase* with:

- *Higher coupons*—because there's more to reinvest.
- *Longer maturities*—because more of the total value of the investment is in the coupon cash flows (and interest on coupon cash flows).

In both cases, the amount of reinvested income will play a bigger role in determining the bond's total return and, therefore, introduce more reinvestment risk. A non-callable zero-coupon bond has no reinvestment risk over its life because there are no cash flows to reinvest prior to maturity.

LOS 67.d: Compute the bond equivalent yield of an annual-pay bond, and compute the annual-pay yield of a semiannual-pay bond.

This LOS requires that you be able to turn a semiannual return into an annual return, and an annual return into a semiannual return.

Example: Comparing bonds with different coupon frequencies

Suppose that Daimler-Chrysler has a semiannual coupon bond trading in the U.S. with a YTM of 6.25 percent, and an annual coupon bond trading in Europe with a YTM = 6.30 percent. Which bond has the greater yield?

Answer:

To determine the answer, we can convert the yield on *the annual-pay bond* to a (semiannual-pay) bond equivalent yield. That is:

$$\text{BEY of an annual-pay bond} = [(1 + \text{annual YTM})^{\frac{1}{2}} - 1] \times 2$$

Thus, the BEY of the 6.30 percent annual-pay bond is:

$$[(1+0.0630)^{0.5} - 1] \times 2 = [1.031 - 1] \times 2 = 0.031 \times 2 = 0.062 = 6.2\%$$

The 6.25 percent semiannual-pay bond provides the better (bond equivalent) yield.

Alternatively, we could convert the YTM of the semiannual-pay bond (which is a bond equivalent yield) to an equivalent annual-pay basis. The equivalent annual yield (EAY—*sometimes known as the effective annual yield*) to the 6.25 percent semiannual-pay YTM is:

$$\text{equivalent annual yield} = \left(1+\frac{0.0625}{2}\right)^2 - 1 = 0.0635 \rightarrow 6.35\%$$

The EAY of the semiannual-pay bond is 6.35 percent, which is greater than the 6.3 percent for the annual-pay bond. Therefore, the semiannual-pay bond has a greater yield as long as we put the yields on an equivalent basis, calculating both as annual yields or calculating both as bond equivalent yields (semiannual yields × 2).

LOS 67.e: Compute the theoretical Treasury spot rate curve, using the method of bootstrapping and given the Treasury par yield curve and compute the value of a bond using spot rates.

Valuing a Bond With Spot Rates

Spot rates are the discount rates for single payments; you can think of them as the discount rates on zero-coupon bonds. The Treasury spot rate curve serves to provide yields for many different maturities of zero-coupon Treasuries. In the previous topic review, we used spot rates to discount the "pieces" of a Treasury bond in demonstrating the arbitrage-free pricing result. An example will demonstrate the method of valuing a bond using spot rates:

Example: Valuing a bond using spot rates

Given the following spot rates (in BEY form):

0.5 years = 4%
1.0 years = 5%
1.5 years = 6%

Calculate the value of a 1.5 year, 8 percent Treasury bond.

Answer:

Simply lay out the cash flows and discount by the spot rates, which are one-half the quoted rates since they are quoted in BEY form.

$$\frac{4}{\left(1+\frac{0.04}{2}\right)^1}+\frac{4}{\left(1+\frac{0.05}{2}\right)^2}+\frac{104}{\left(1+\frac{0.06}{2}\right)^3} = 102.9 \text{ or, with the TVM calculator function,}$$

N = 1; PMT = 0; I/Y = 2; FV = 4; CPT → PV = –3.92
N = 2; PMT = 0; I/Y = 2.5; FV = 4; CPT → PV = –3.81
N = 3; PMT = 0; I/Y = 3; FV = 104; CPT → PV = –95.17

Add these values together to get 102.9.

The Method of Bootstrapping

The par yield curve gives the YTMs of bonds currently trading near their par values (YTM ≈ coupon rate) for various maturities. Here, we need to use these yields to get the theoretical Treasury spot rate curve by a process called bootstrapping.

The method of bootstrapping can be a little confusing, so let's first get the main idea and then go through a more realistic and detailed example. The general idea is that we will solve for spot rates by knowing the prices of coupon bonds. We always know one spot rate to begin with and then calculate the spot rate for the next longer period. When we know two spot rates, we can get the third based on the market price of a bond with three cash flows by using the spot rates to get the present values of the first two cash flows.

As an example of this method, consider that we know the prices and yields of three annual-pay bonds as shown in Figure 2. All three bonds are trading at par or $1,000.

Figure 2: Prices and Yield for Three Annual-Pay Bonds

Maturity	*Coupon*	*Yield*	*Price*
1 year	3%	3%	$1,000
2 years	4%	4%	$1,000
3 years	5%	5%	$1,000

Since the one-year bond makes only one payment (it's an annual-pay bond) of $1,030 at maturity, the one-year spot rate is 3%, the yield on this single payment. The two-year bond makes two payments, a $40 coupon in one year and a $1,040 payment at maturity in two years. Since the spot rate to discount the 2-year bond's first cash

flow is 3%, and since we know that the sum of the present values of the bond's cash flows must equal its (no arbitrage) price of $1,000, we can write:

$$\frac{40}{1.03}+\frac{1{,}040}{(1+2\text{-year spot rate})^2}=\$1{,}000$$

Based on this we can solve for the two-year spot rate as follows:

1. $$\frac{1{,}040}{(1+2\text{-year spot})^2}=1{,}000-\frac{40}{1.03}=1{,}000-38.83=961.17$$

2. $$\frac{1{,}040}{961.17}=(1+2\text{-year spot})^2=1.082$$

3. $$2\text{-year spot}=(1.082)^{\frac{1}{2}}-1=0.04019=4.019\%$$

Now that we have both the 1-year and 2-year spot rates, we can use the cash flows and price of the 3-year bond to write:

$$\frac{50}{1.03}+\frac{50}{(1.0419)^2}+\frac{1{,}050}{(1+3\text{-year spot})^3}=1{,}000$$

And solve for the 3-year spot rate:

$$1{,}000-\frac{50}{1.03}-\frac{50}{(1.0419)^2}=\frac{1{,}050}{(1+3\text{-year spot})^3}$$

$$1{,}000-48.54-46.06=\frac{1{,}050}{(1+3\text{-year spot})^3}$$

$$905.40=\frac{1{,}050}{(1+3\text{-year spot})^3}$$

$$\left(\frac{1{,}050}{905.40}\right)^{1/3}-1=3\text{-year spot}=0.05063=5.063\%$$

So we can state that:

$$\frac{50}{1.03}+\frac{50}{(1.0419)^2}+\frac{1{,}050}{(1.05063)^3}=\$1{,}000$$

We have just solved for the 2-year and 3-year spot rates by the method of bootstrapping.

In practice, Treasury bonds pay semiannually and their YTMs are semiannual-pay YTMs. The next example illustrates the method of bootstrapping when coupons are paid semiannually.

Consider the yields on coupon Treasury bonds trading at par given in Figure 3. YTM for the bonds is expressed as a bond equivalent yield (semiannual-pay YTM).

Figure 3: Par Yields for Three Semiannual-Pay Bonds

Maturity	*YTM*	*Coupon*	*Price*
6 mos.	5%	5%	100
1 year	6%	6%	100
18 mos.	7%	7%	100

The bond with six months left to maturity has a semiannual discount rate of $0.05/2 = 0.025 = 2.5\%$ or 5 percent on an annual BEY basis. Since this bond will only make one payment of 102.5 in six months, the YTM is the spot rate for cash flows to be received six months from now.

The bootstrapping process proceeds from this point using the fact that the 6-month annualized spot rate is 5 percent together with the price/YTM information on the 1-year bond. We will use the formula for valuing a bond using spot rates that we covered earlier.

Noting that the 1-year bond will make two payments, one in six months of 3.0 and one in one year of 103.0, and that the appropriate spot rate to discount the coupon payment (which comes six months from now), we can write:

$$\frac{3}{1.025}+\frac{103}{\left(1+S_{1.0}/2\right)^2}=100, \text{ where } S_{1.0} \text{ is the annualized 1-year spot rate, and solve for } S_{1.0}/2 \text{ as:}$$

$$\frac{103}{\left(1+S_{1.0}/2\right)^2}=100-\frac{3}{1.025}=100-2.927=97.073$$

$$\frac{103}{97.073}=\left(1+S_{1.0}/2\right)^2 \text{ so } \sqrt{\frac{103}{97.073}}-1=S_{1.0}/2=0.030076 \text{ and } S_{1.0}=2\times0.030076=0.060152=6.0152\%$$

Now that we have the 6-month and 1-year spot rates, we can use this information and the price of the 18-month bond to set the bond price equal to the value of the bond's cash flows as:

$$\frac{3.5}{1.025}+\frac{3.5}{(1.030076)^2}+\frac{103.5}{\left(1+S_{1.5}/2\right)^3}=100, \text{ where } S_{1.5} \text{ is the annualized 1.5-year spot rate, and solve for } S_{1.5}/2$$

$$\frac{103.5}{\left(1+S_{1.5}/2\right)^3}=100-\frac{3.5}{1.025}-\frac{3.5}{(1.030076)^2}=100-3.415-3.30=93.285$$

$$\frac{103.5}{93.285}=\left(1+S_{1.5}/2\right)^3 \text{ so } \left(\frac{103.5}{93.285}\right)^{\frac{1}{3}}-1=S_{1.5}/2=0.035244 \text{ and } S_{1.5}=2\times0.035244=0.070488=7.0488\%$$

To summarize the method of bootstrapping spot rates from the par yield curve:

1. Begin with the 6-month spot rate.

2. Set the value of the 1-year bond equal to the present value of the cash flows with the 1-year spot rate divided by 2 as the only unknown.

3. Solve for the 1-year spot rate.

4. Use the 6-month and 1-year spot rates and equate the present value of the cash flows of the 1.5 year bond equal to its price, with the 1.5 year spot rate as the only unknown.

5. Solve for the 1.5-year spot rate.

LOS 67.f: Explain the limitations of the nominal spread and differentiate among the nominal spread, the zero-volatility spread, and the option-adjusted spread for a bond with an embedded option, and explain the option cost.

The **nominal spread** is the simplest of the spread measures to use and to understand. It is simply an issue's YTM minus the YTM of a Treasury security of similar maturity. Therefore, the use of the nominal spread suffers from the same limitations as the YTM. YTM uses a single discount rate to value the cash flows, so it *ignores the shape of the spot yield curve*. In fact, YTM for a coupon bond is theoretically correct only to the extent that the spot rate curve is flat. When the yield curve is steep, nominal yield spreads are affected and nominal spreads for amortizing securities are the most affected.

The Zero-Volatility Spread and the Option-Adjusted Spread

One way to get a bond's nominal spread to Treasuries would be to add different amounts to the yield of a comparable Treasury bond, and value the bond with those YTM's. The amount added to the Treasury yield that produces a bond value equal to the market price of the bond must be the nominal yield spread.

This may seem like an odd way to get the spread, but it makes sense when you see how the **zero-volatility spread**, or static spread, is calculated. The zero-volatility spread (Z-spread) is the equal amount that we must add to each rate on the Treasury spot yield curve in order to make the present value of the risky bond's cash flows equal to its market price. Instead of measuring the spread to YTM, the *zero-volatility spread* measures the spread to Treasury spot rates necessary to produce a spot rate curve that "correctly" prices a risky bond (i.e., produces its market price).

For a risky bond, the value obtained from discounting the expected cash flows at Treasury spot rates will be too high because the Treasury spot rates are lower than those appropriate for a risky bond. In order to value it correctly, we have to increase each of the Treasury spot rates by some equal amount so that the present value of the risky bond's cash flows discounted at the (increased) spot rates equals the market value of the bond. The following example will illustrate the process for calculating the Z-spread.

Example: Zero-volatility spread

1-, 2-, and 3-year spot rates on Treasuries are 4 percent, 8.167 percent, and 12.377 percent, respectively. Consider a 3-year, 9 percent annual coupon corporate bond trading at 89.464. The YTM = 13.50% and the YTM of a 3-year Treasury is 12 percent. **Compute** the nominal spread and the zero-volatility spread of the corporate bond.

Answer:

The *nominal spread* is:

$$\text{nominal spread} = \text{YTM}_{\text{Bond}} - \text{YTM}_{\text{Treasury}} = 13.50 - 12.00 = 1.50\%$$

To compute the *Z-spread,* set the present value of the bond's cash flows equal to today's market price. Discount each cash flow at the appropriate zero-coupon bond spot rate *plus* a fixed spread = ZS. Solve for ZS in the following equation and you have the *Z*-spread:

$$89.464 = \frac{9}{(1.04 + \text{ZS})^1} + \frac{9}{(1.08167 + \text{ZS})^2} + \frac{109}{(1.12377 + \text{ZS})^3} \Rightarrow \text{ZS} = 1.67\% \text{ or } 167 \text{ basis points}$$

Note that this spread is found by trial-and-error. In other words, pick a number "ZS," plug it into the right-hand side of the equation, and see if the result equals 89.464. If the right-hand side equals the left, then you have found the *Z*-spread. If not, pick another "ZS" and start over.

Professor's Note: This is not a calculation you are expected to make; this example is to help you understand how a Z-spread differs from a nominal spread.

The **option-adjusted spread** (OAS) measure is used when a bond has embedded options. A callable bond, for example, must have a greater yield than an identical option-free bond, and a greater nominal spread or *Z*-spread. Without accounting for the value of the options, these spread measures will suggest the bond is a great value when, in fact, the additional yield is compensation for call risk. Loosely speaking, the *option-adjusted spread* takes the option yield component out of the *Z*-spread measure; the option-adjusted spread is the spread to the Treasury spot rate curve that the bond would have if it were option-free. The OAS is the spread for non-option characteristics like credit risk, liquidity risk, and interest rate risk. The actual method of calculation is reserved for Level 2; for our purposes, however, an understanding of what the OAS is will be sufficient.

Option Cost in Percent

If we calculate an option-adjusted spread for a callable bond, it will be less than the bond's *Z*-spread. The difference is the extra yield required to compensate for the call option. Calling that extra yield the **option cost,** we can write:

Z-spread – OAS = option cost in %

Example: Cost of an embedded option

Suppose you learn that a bond is callable and that it has an OAS of 135bp. You also know that similar bonds have a *Z*-spread of 167 basis points. **Compute** cost of the embedded option.

Answer:

The option cost = Z-spread – OAS = 167 – 135 = 32 basis points.

For embedded short calls (e.g., callable bonds): option cost > 0 (you receive compensation for writing the option to the issuer) → OAS < Z-spread. In other words, you *require more yield on the callable bond* than for an option-free bond.

For embedded puts (e.g., putable bonds), option cost < 0 (i.e., you must pay for the option) → OAS > Z-spread. In other words, you *require less yield on the putable bond* than for an option-free bond.

LOS 67.g: Explain a forward rate, and compute the value of a bond using forward rates, explain and illustrate the relationship between short-term forward rates and spot rates, and compute spot rates given forward rates, and forward rates given spot rates.

A **forward rate** is a borrowing/lending rate for a loan to be made at some future date. The notation used must identify both the length of the lending/borrowing period and when in the future the money will be loaned/borrowed. Thus ${}_1f_1$ is the rate for a 1-year loan one year from now and ${}_1f_2$ is the rate for a 1-year loan to be made two years from now, etc. Rather than introduce a separate notation, we can represent the current 1-year rate as ${}_1f_0$. To get the present values of a bond's expected cash flows we need to discount each cash flow by the forward rates for the number of periods until it is received.

Example: Computing a bond value using forward rates

The current 1-year rate (${}_1f_0$) is 4 percent and the 1-year forward rate for lending from time = 1 to time = 2 is ${}_1f_1 = 5\%$, and the 1-year forward rate for lending from time = 2 to time = 3 is ${}_1f_2 = 6\%$. Value a 3-year annual-pay bond with a 5 percent coupon and a par value of $1,000.

Answer:

$$\text{bond value} = \frac{50}{1+{}_1f_0} + \frac{50}{(1+{}_1f_0)(1+{}_1f_1)} + \frac{1{,}050}{(1+{}_1f_0)(1+{}_1f_1)(1+{}_1f_2)} =$$

$$\frac{50}{1.04} + \frac{50}{(1.04)(1.05)} + \frac{1{,}050}{(1.04)(1.05)(1.06)} = \$1{,}000.98$$

Professor's Note: If you think this looks a little like valuing a bond using spot rates, as we did for arbitrage-free valuation, you are right. The discount factors are equivalent to spot rate discount factors.

The Relationship Between Short-Term Forward Rates and Spot Rates

The idea here is that *borrowing for three years at the 3-year rate or borrowing for 1-year periods, three years in succession, should have the same cost.*

This relation is illustrated as $(1+S_3)^3 = (1 + {}_1f_0)(1+{}_1f_1)(1+{}_1f_2)$ and the reverse as

$S_3 = \left[(1 + {}_1f_0)(1+{}_1f_1)(1+{}_1f_2)\right]^{\frac{1}{3}} - 1$, which is the geometric mean we covered in Study Session 2.

Example: Computing spot rates from forward rates

If the current 1-year rate is 2 percent, the 1-year forward rate (${}_1f_1$) is 3 percent and the 2-year forward rate (${}_1f_2$) is 4 percent, what is the 3-year spot rate?

Answer:

$$S_3 = \left[(1.02)(1.03)(1.04)\right]^{\frac{1}{3}} - 1 = 2.997\%$$

This can be interpreted to mean that a dollar compounded at 2.997 percent for three years would produce the same ending value as a dollar that earns compound interest of 2 percent the first year, 3 percent the next year, and 4 percent for the third year.

Professor's Note: You can get a very good approximation of the 3-year spot rate with the simple average of the forward rates. In the previous example we got 2.997% and the simple average of the three annual rates is $\frac{2+3+4}{3} = 3\%$!

Forward Rates Given Spot Rates

We can use the same relationship we used to calculate spot rates from forward rates to calculate forward rates from spot rates.

Our basic relation between forward rates and spot rates (for two periods) is:

$$(1+S_2)^2 = (1+{}_1f_0)(1+{}_1f_1)$$

Which, again, tells us that an investment has the same expected yield (borrowing has the same expected cost) whether we invest (borrow) for two periods at the two-period spot rate, S_2, or for one period at the current rate, S_1, and for the next period at the expected forward rate, ${}_1f_1$. Clearly, given two of these rates, we can solve for the other.

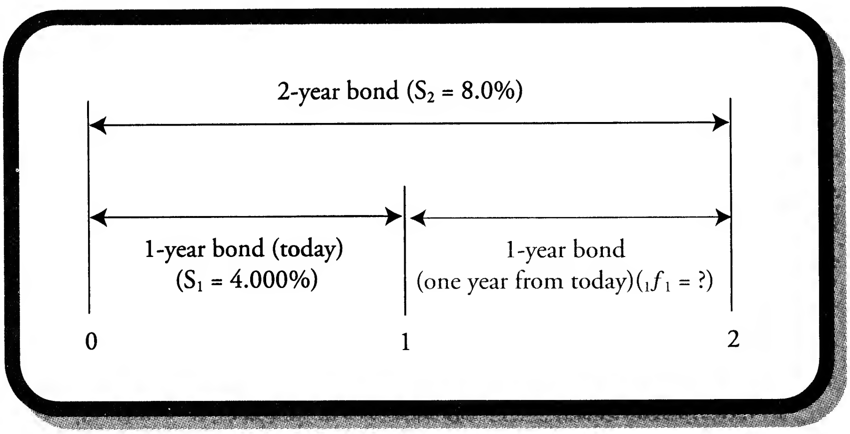
Example: Computing a forward rate from spot rates

The two-period spot rate, S_2, is 8 percent and the current one-period (spot) rate is 4 percent (this is both S_1 and ${}_1f_0$). **Calculate** the forward rate for one period, one period from now, ${}_1f_1$.

Answer:

Figure 4 illustrates the problem.

Figure 4: Finding a Forward Rate



From our original equality, $(1+S_2)^2 = (1+S_1)(1+{}_1f_1)$, we can get $\frac{(1+S_2)^2}{(1+S_1)} - 1 = {}_1f_1$ or, since we know that both choices have the same payoff in two years:

$$(1.08)^2 = (1.04)(1 + {}_1f_1)$$

$$(1 + {}_1f_1) = \frac{(1.08)^2}{(1.04)}$$

$${}_1f_1 = \frac{(1.08)^2}{(1.04)} - 1 = \frac{1.1664}{1.04} - 1 = 12.154\%$$

In other words, investors are willing to accept 4.0 percent on the 1-year bond today (when they could get 8.0 percent on the 2-year bond today) only because they can get 12.154 percent on a 1-year bond one year from today. This future rate that can be locked in today is a *forward rate*.

Similarly, we can back other forward rates out of the spot rates. We know that:

$$(1 + S_3)^3 = (1 + S_1)(1 + {}_1f_1)(1 + {}_1f_2)$$

And that:

$$(1 + S_2)^2 = (1 + S_1)(1 + {}_1f_1), \text{ so we can write } (1 + S_3)^3 = (1 + S_2)^2(1 + {}_1f_2)$$

This last equation says that investing for three years at the 3-year spot rate should produce the same ending value as investing for two years at the 2-year spot rate and then for a third year at ${}_1f_2$, the 1-year forward rate, two years from now.

Solving for the forward rate, ${}_1f_2$, we get:

$$\frac{(1+S_3)^3}{(1+S_2)^2} - 1 = {}_1f_2$$

Example: Forward rates from spot rates

Let's extend the previous example to three periods. The current 1-year spot rate is 4.0 percent, the current 2-year spot rate is 8.0 percent, and the current 3-year spot rate is 12.0 percent. **Calculate** the 1-year forward rates one and two years from now.

Answer:

We know the following relation must hold:

$$(1+ S_2)^2 = (1 + S_1)(1 + {}_1f_1)$$

We can use it to solve for the 1-year forward rate one year from now:

$$(1.08)^2 = (1.04)(1 + {}_1f_1), \text{ so } {}_1f_1 = \frac{(1.08)^2}{(1.04)} - 1 = 12.154\%$$

We also know that the relations:

$$(1 + S_3)^3 = (1 + S_1)(1 + {}_1f_1)(1 + {}_1f_2)$$

and, equivalently $(1 + S_3)^3 = (1 + S_2)^2(1 + {}_1f_2)$ must hold.

Substituting values for S_3 and S_2, we have:

$$(1.12)^3 = (1.08)^2 \times (1 + {}_1f_2)$$

so that the 1-year forward rate two years from now is:

$${}_1f_2 = \frac{(1.12)^3}{(1.08)^2} - 1 = 20.45\%$$

To verify these results, we can check our relations by calculating:

$$S_3 = \left[(1.04)(1.12154)(1.2045)\right]^{\frac{1}{3}} - 1 = 12.00\%$$

This may all seem a bit complicated, but the basic relation, that borrowing for successive periods at one-period rates should have the same cost as borrowing at multiperiod spot rates, can be summed up as:

$(1 + S_2)^2 = (1 + S_1)(1 + {}_1f_1)$ for two periods, and $(1 + S_3)^3 = (1 + S_2)^2(1 + {}_1f_2)$ for three periods.

Professor's Note: Simple averages also give decent approximations for calculating forward rates from spot rates. In the above example, we had spot rates of 4% for 1 year and 8% for two years. 2 years at 8% is 16%, so if the first year rate is 4%, the second year rate is close to 16 – 4 = 12% (actual is 12.154). Given a 2 year spot rate of 8% and a 3 year spot rate of 12%, we could approximate the 1-year forward rate from time 2 to time 3 as (3 × 12) – (2 × 8) = 20. 20 may be close enough (actual is 20.45) to answer a multiple choice question and, in any case, serves as a good check to make sure the exact rate you calculate is reasonable.

KEY CONCEPTS

1. There are three sources of return to a coupon bond: coupon interest payments, reinvestment income on the coupons, and capital gain or loss on the principal value.
2. Yield to maturity (YTM) can be calculated on a semiannual or annual basis and when calculated on a semiannual basis is termed a bond equivalent yield (BEY).
3. YTM is not the realized yield on an investment unless the reinvestment rate is equal to the YTM.
4. Other important yield measures are the current yield, yield to call, yield to put, and (monthly) cash flow yield.
5. Reinvestment risk is higher when the coupon rate is greater (maturity held constant) and when the bond has longer maturity (coupon rate held constant).
6. The spot rate curve gives the rates to discount each separate cash flow based on the time when it will be received. YTM is an IRR measure that does not account for the shape of the yield curve.
7. There are three commonly used yield spread measures:
   - nominal spread: bond YTM – Treasury YTM.
   - zero-volatility spread (*Z*-spread or static spread): spread to the spot yield curve.
   - option-adjusted spread (OAS): spread to the spot yield curve after adjusting for the effects of embedded options, reflects the spread for credit risk and illiquidity.
8. The *Z*-spread – OAS = option cost in %.
   - For callable bonds: Z-spread > OAS and option cost > 0.
   - For putable bonds: Z-spread < OAS and option cost < 0.
9. Forward rates are current lending/borrowing rates for loans to be made in future periods.
10. Spot rates for a maturity of "*N*" periods are the geometric mean of compound forward rates over the *N* periods.

Exam Flashbacks

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #4 from '94 actual exam.

Zello Corporation's $1,000 par value bond sells for $960, matures in five years, and has a 7% coupon rate paid semiannually. What is the bond's current yield?
A. 7.0%.
B. 7.3%.
C. 8.0%.
D. Insufficient information provided.

Exam Flashback # 2

Source: Question #80 from '91 actual exam.

Find the yield-to-maturity of a 20-year zero-coupon bond that is selling for $372.50 (use annual compounding and assume the issue has a maturity value of $1,000).
A. 5.1%.
B. 8.8%.
C. 10.1%.
D. 13.4%.

Exam Flashback # 3

Source: Question #101 from '92 and '96 actual exams, and '97–'98 sample exams.

Yield-to-maturity and current yield on a bond are equal:
A. when market interest rates begin to level off.
B. if the bond sells at a price in excess of its par value.
C. when the expected holding period is greater than one year.
D. if the coupon and market interest rate are equal.

Exam Flashback # 4

Source: Question #63 from '91 actual exam.

A 10% annual pay, 20 year bond is priced at $850.61 to yield 12%; if it paid interest semiannually (rather than annually), but continued to be priced $850.61, the resulting yield-to-maturity would be:
A. less than 12%.
B. more than 12%.
C. 12%.
D. cannot be determined.

Exam Flashback # 5

Source: Question #4 from '94 actual exam.

Zello Corporation's $1,000 par value bond sells for $960, matures in five years, and has a 7% coupon rate paid semiannually. What is the bond's yield to maturity?
A. 7.0%.
B. 7.3%.
C. 8.0%.
D. 8.1%.

Exam Flashback # 6

Source: Question #70 from '91 actual exam.

For a callable bond, yield-to-call is a more conservative measure of yield whenever:
A. the bond is priced at or above its call price.
B. the bond is trading for more than par but less than the call price.
C. the bond is trading at less than par.
D. none of the above.

Exam Flashback # 7

Source: Question #101 from '00–'03 sample exams.

An analyst gathered the following spot rates:

Time (years)	*Annual Spot Rate*
1	15.0%
2	12.5%
3	10.0%
4	7.5%

The one-year forward rate two years from now is *closest* to:
A. – 4.91%.
B. 5.17%.
C. 10.05%.
D. 15.74%.

Exam Flashback # 8

Source: Question #93 from 1994 actual exam.

The 6-month Treasury bill spot rate is 4.0 percent, and the 1-year Treasury bill spot rate is 5.0 percent. The implied 6-month forward rate 6 months from now is:
A. 3.0 percent.
B. 4.5 percent.
C. 5.5 percent.
D. 6.0 percent.

Concept Checkers: Yield Measures, Spot Rates, and Forward Rates

Use the following data to answer Questions 1 through 4.

An analyst observes a Widget & Co. 7.125 percent, 4-year, *semiannual-pay* bond trading at 102.347 percent of par (where par = $1,000). The bond is callable at 101 in two years, and putable at 100 in two years.

1. What is the bond's current yield?
   A. 6.962%.
   B. 7.500%.
   C. 7.426%.
   D. 7.328%.

2. What is the bond's yield to maturity?
   A. 3.225%.
   B. 6.450%.
   C. 6.334%.
   D. 5.864%.

3. What is the bond's yield to call?
   A. 3.167%.
   B. 5.664%.
   C. 6.334%.
   D. 5.864%.

4. What is the bond's yield to put?
   A. 2.932%.
   B. 6.450%.
   C. 4.225%.
   D. 5.864%.

5. Based on semiannual compounding, what would the yield to maturity (YTM) be on a 15-year, zero-coupon, $1,000 par value bond that's currently trading at $331.40?
   A. 3.750%.
   B. 5.151%.
   C. 7.500%.
   D. 7.640%.

6. An analyst observes a bond with an *annual* coupon that's being priced to yield 6.350 percent. What is this issue's bond equivalent yield?
   A. 3.175%.
   B. 3.126%.
   C. 6.252%.
   D. 6.172%.

7. An analyst determines that the cash flow yield of GNMA Pool 3856 is 0.382 percent *per month*. What is the bond equivalent yield?
   A. 9.582%.
   B. 9.363%.
   C. 4.682%.
   D. 4.628%.

8. For the YTM to equal the actual compound return an investor realizes on an investment in a coupon bond, we must assume all of the following **EXCEPT**:
   A. cash flows will be paid as promised.
   B. the bond will not be sold at a capital loss.
   C. cash flows will be reinvested at the YTM rate.
   D. The bond will be held until maturity.

9. The 4-year spot rate is 9.45 percent, and the 3-year spot rate is 9.85 percent. What is the 1-year forward rate three years from today?
   A. 0.400%.
   B. 9.850%.
   C. 8.258%.
   D. 11.059%.

10. An investor purchases a bond that is putable at the option of the holder. The option has value. He has calculated the *Z*-spread as 223 basis points. The option-adjusted spread will be:
    A. equal to 223 basis points.
    B. less than 223 basis points.
    C. greater than 223 basis points.
    D. It is not possible to determine from the data given.

Use the following data to answer Questions 11 and 12.

Given:

Current 1-year rate = 5.5%
${}_1f_1 = 7.63\%$
${}_1f_2 = 12.18\%$
${}_1f_3 = 15.5\%$

11. The value of a 4-year, 10 percent annual-pay, $1,000 par value bond would be *closest* to:
    A. $844.55.
    B. $995.89.
    C. $1,009.16.
    D. $1,085.62.

12. Using annual compounding, the value of a 3-year, zero-coupon, $1,000 par value bond would be:
    A. $708.
    B. $785.
    C. $852.
    D. $948.

13. A bond's nominal spread, zero-volatility spread, and option-adjusted spread will all be equal for a coupon bond if:
    A. the coupon is low and the yield curve is flat.
    B. the yield curve is flat and the bond is not callable.
    C. the bond is option free.
    D. the coupon is high, the yield curve is flat, and the bond has no embedded options.

14. The zero-volatility spread will be zero:
    A. for any bond that is option-free.
    B. if the yield curve is flat.
    C. for a zero-coupon bond.
    D. for an on-the-run Treasury bond.

ANSWERS – EXAM FLASHBACKS

1. **B** CY = \$ coupons / market price. Hence, CY = 70 / 960 = 7.29.

2. **A** N = 20; PV = –\$372.50; FV = \$1,000; PMT=0; CPT → I/Y = 5.06 = 5.1%.

3. **D** When the coupon rate and market interest rate are equal; the bond sells at its par value. In this case, the current yield is (annual \$ coupon) / \$1,000 = coupon rate. The YTM is also called the market interest rate.

4. **A** Logic: Since the price didn't change, and semiannual rates cause money to grow faster, the new rate is less than 12%. With your financial calculator: PV = –\$850.61;

   FV = \$1,000; PMT = \$50; N = 20 × 2 = 40; CPT → I/Y = 5.992 or double 11.984.

5. **C** FV = \$1,000; PV = –\$960; N = 5 × 2 = 10; PMT = 70/2 = 35; CPT → I/Y = 3.99. This is a semiannual rate, so double to get the annualized yield = 7.98.

6. **A** When the bond is priced above the call price, the embedded option to call the bond is "in-the-money." Hence, it is more likely that the firm will call the bonds away from investors and the use of the call price and the date to first call is more appropriate.

7. **B** $$f_2 = \frac{(1+S_3)^3}{(1+S_2)^2} - 1$$

   $$= \frac{(1.10)^3}{(1.125)^2} - 1 = 0.0517 \text{ or } 5.17\%$$

   Note: ${}_1f_2$ denotes the 1-year forward rate two years from today.

8. **D** Treating the given rates as BEY (twice the semiannual rate) we have $\frac{1.025^2}{1.02} - 1 = 3.0025\%$ for the second six months, which is 2 × 3.0025% = 6.005% when expressed as a BEY.

ANSWERS – CONCEPT CHECKERS: YIELD MEASURES, SPOT RATES, AND FORWARD RATES

1. **A** $$\text{Current yield} = \frac{71.25}{1{,}023.47} = 0.06962, \text{ or } 6.962\%.$$

2. **B** $$1{,}023.47 = \sum_{t=1}^{8} \frac{35.625}{(1+\text{YTM}/2)^t} + \frac{1{,}000}{(1+\text{YTM}/2)^8} \Rightarrow \text{YTM} = 6.450\%$$

   N = 8; FV = 1,000; PMT = 35.625; PV = –1,023.47 → CPT I/Y = 3.225 × 2 = 6.45%

3. **C** $$1{,}023.47 = \sum_{t=1}^{4} \frac{35.625}{(1+\text{YTC}/2)^t} + \frac{1{,}010}{(1+\text{YTC}/2)^4} \Rightarrow \text{YTC} = 6.334\%$$

   N = 4; FV = 1,010; PMT = 35.625; PV = –1,023.47; CPT → I/Y = 3.167 × 2 = 6.334%

4. **D** $$1{,}023.47 = \sum_{t=1}^{4} \frac{35.625}{(1+\text{YTP}/2)^t} + \frac{1{,}000}{(1+\text{YTP}/2)^4} \Rightarrow \text{YTP} = 5.864\%$$

   N = 4; FV = 1,000; PMT = 35.625; PV = –1,023.47; CPT → I/Y = 2.932 × 2 = 5.864%

5. **C** $\left[\left(\frac{1{,}000}{331.40}\right)^{\frac{1}{30}} - 1\right] \times 2 = 7.5\%$ or,

Solving with a financial calculator:

N = 30; FV = 1,000; PMT = 0; PV = –331.40; CPT → I/Y = 3.750 × 2 = 7.500%

6. **C** $\text{Bond equivalent yield} = \left([1+\text{EAY}]^{\frac{1}{2}} - 1\right) \times 2 = \left([1.0635]^{\frac{1}{2}} - 1\right) \times 2 = 6.252\%$

7. **D** $\text{Bond equivalent yield} = \left([1+\text{CFY}]^{6} - 1\right) \times 2 = \left([1.00382]^{6} - 1\right) \times 2 = 4.628\%$

8. **B** For a bond purchased at a premium to par value, a decrease in the premium over time (a capital loss) is already factored into the calculation of YTM.

9. **C** $(1.0945)^4 = (1.0985)^3 \times (1 + {}_1f_3)$

$$\frac{(1.0945)^4}{(1.0985)^3} - 1 = {}_1f_3 = 8.258\%$$

10. **C** For embedded puts (e.g., putable bonds): option cost < 0, ⇒ OAS > Z-spread.

11. **C** Spot rates: $S_1 = 5.5\%$.

$$S_2 = [(1.055)(1.0763)]^{\frac{1}{2}} - 1 = 6.56\%$$

$$S_3 = [(1.055)(1.0763)(1.1218)]^{\frac{1}{3}} - 1 = 8.39\%$$

$$S_4 = [(1.055)(1.0763)(1.1218)(1.155)]^{\frac{1}{4}} - 1 = 10.13\%$$

Bond value:

N=1; FV=100; I/Y=5.5; CPT→PV	=	94.79
N=2; FV=100; I/Y=6.56; CPT→PV	=	88.07
N=3; FV=100; I/Y=8.39; CPT→PV	=	78.53
N=4; FV=1,100; I/Y=10.13; CPT→PV	=	747.77
Total:		$1,009.16

12. **B** Find the spot rate for 3-year lending:

$$S_3 = [(1.055)(1.0763)(1.1218)]^{\frac{1}{3}} - 1 = 8.39\%$$

Value of the bond: N = 3; FV=1,000; I/Y=8.39; CPT → PV = 785.29,

or

$$\frac{\$1{,}000}{(1.055)(1.0763)(1.1218)} = \$785.05$$

13. **D** If the yield curve is flat, the nominal spread and the Z-V spread are equal. If the bond is option-free, the Z-V spread and OAS are equal. The coupon rate is not relevant.

14. **D** A Treasury bond is the best answer. The Treasury spot yield curve will correctly price an on-the-run Treasury bond at its arbitrage-free price, so the Z-V spread is zero.

Introduction to the Measurement of Interest Rate Risk

Exam Focus

This topic review is all about the relation of yield changes and bond price changes, primarily based on the concepts of duration and convexity. There is really nothing in this study session that can be safely ignored; the calculation of duration, the use of duration, and the limitations of duration as a measure of bond price risk are all important. You should work to understand what convexity is and its relation to the interest rate risk of fixed income securities. There are two important formulas: the formula for effective duration and the formula for estimating the price effect of a yield change based on both duration and convexity. Finally, you should get comfortable with how and why the convexity of a bond is affected by the presence of embedded options.

LOS 68.a: Distinguish between the full valuation approach (the scenario analysis approach) and the duration/convexity approach for measuring interest rate risk, and explain the advantage of using the full valuation approach.

The **full valuation or scenario analysis approach** to measuring interest rate risk is based on applying the valuation techniques we have learned for a given change in the yield curve (i.e., for a given *interest rate scenario*). For a single option-free bond this could be simply, "if the YTM increases by 50 bp or 100 bp what is the impact on the value of the bond?" More complicated scenarios can be used as well, such as the effect on the bond value of a steepening of the yield curve where long term rates increase more than short term rates. If our valuation model is good, the exercise is straightforward: plug in the rates described in the interest rate scenario(s) and see what happens to the values of the bonds. If the valuation model used is sufficiently good, this is the theoretically preferred approach. Applied to a portfolio of bonds, one bond at a time, we can get a very good idea of how different changes in interest rates will affect the value of the portfolio.

The **duration/convexity approach** provides an approximation of the actual interest rate sensitivity of a bond or bond portfolio. Its main advantage is its simplicity compared to the full valuation approach. The full valuation approach can get quite complex and time consuming for a portfolio of more than a few bonds, especially if some of the bonds have more complex structures, such as call provisions. As we will see shortly, limiting our scenarios to parallel yield curve shifts and "settling" for an estimate of interest rate risk allows us to use the summary measures, duration and convexity. This greatly simplifies the process of estimating the value impact of overall changes in yield. Compared to the duration/convexity approach, the full valuation approach is more precise and can be used to evaluate the price effects of more complex interest rate scenarios than the duration-convexity approach, which strictly speaking is appropriate only for estimating the effects of parallel yield curve shifts.

LOS 68.b: Compute the interest rate risk exposure of a bond position or of a bond portfolio, given a change in interest rates.

In Figure 1, the full valuation approach is illustrated for Bond X, for Bond Y, and for a portfolio containing positions in both Bond X and Bond Y.

Consider two option-free bonds. Bond X is an 8 percent annual-pay bond with 5 years to maturity, priced at 108.4247 to yield 6 percent. (N = 5; PMT = 8.00; FV = 100; I/Y = 6.00%; CPT → PV = –108.4247).

Bond Y is a 5 percent annual-pay bond with 15 years to maturity, priced at 81.7842 to yield 7 percent.

Assume a $10 million face-value position in each bond and two scenarios. The first scenario is a parallel shift in the yield curve of +50 basis points and the second scenario is a parallel shift of +100 basis points. Note that the bond price of 108.4247 is the price per $100 of par value. With $10 million of par value bonds, the market value will be $10.84247 million.

Figure 1: The Full Valuation Approach

		Market Value of:			
Scenario	*Yield Δ*	*Bond X (in millions)*	*Bond Y (in millions)*	*Portfolio*	*Portfolio Value Δ%*
Current	+0 bp	$10.84247	$8.17842	$19.02089	
1	+50 bp	$10.62335	$7.79322	$18.41657	–3.18%
2	+100 bp	$10.41002	$7.43216	$17.84218	–6.20%

N = 5; PMT = 8; FV = 100; I/Y = 6% + 0.5%; CPT → PV = –106.2335
N = 5; PMT = 8; FV = 100; I/Y = 6% + 1%; CPT → PV = –104.1002
N = 15; PMT = 5; FV = 100; I/Y = 7% + 0.5%; CPT → PV = –77.9322
N = 15; PMT = 5; FV = 100; I/Y = 7% + 1%; CPT → PV = –74.3216
Portfolio value change 50 bp: (18.41657 – 19.02089) / 19.02089 = –0.03177 = –3.18%
Portfolio value change 100 bp: (17.84218 – 19.02089) / 19.02089 = –0.06197 = –6.20%

It's worth noting that, on an individual bond basis, the effect of an increase in yield on the bonds' values is less for Bond X than for Bond Y (i.e., with a 50 bp increase in yields, the value of Bond X falls by 2.02 percent while the value of Bond Y falls by 4.71 percent; and with a 100 bp increase, X falls by 3.99 percent while Y drops by 9.12 percent). This, of course, is totally predictable since Bond Y is a longer-term bond and has a lower coupon—both of which mean more interest rate risk.

Professor's Note: Let's review the effects of bond characteristics on duration (price sensitivity). Holding other characteristics the same, we can state:

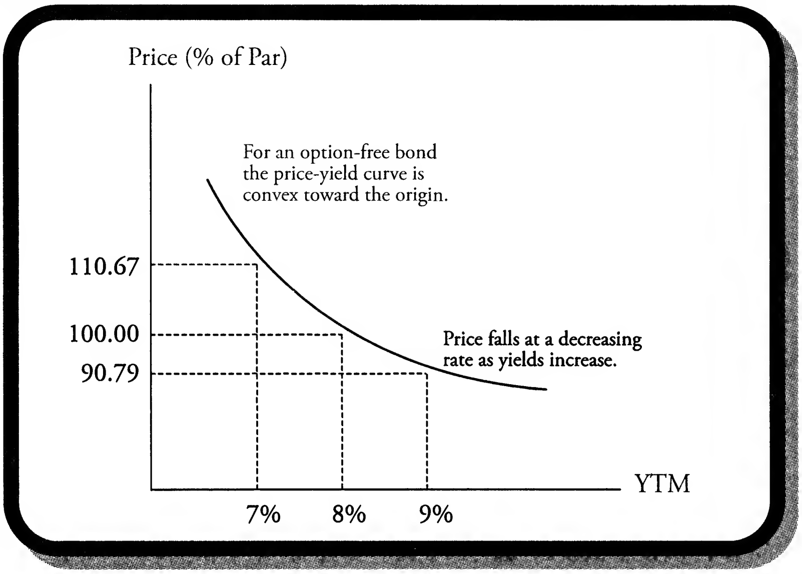
- *Higher (lower) coupon means lower (higher) duration.*
- *Longer (shorter) maturity means higher (lower) duration.*
- *Higher (lower) market yield means lower (higher) duration.*

Finance professors love to test these relations. (See Exam Flashbacks #1–#3.)

LOS 68.c: Demonstrate the price volatility characteristics for option-free bonds when interest rates change (including the concept of "positive convexity"), the price volatility characteristics of callable bonds and prepayable securities when interest rates change (including the concept of "negative convexity"), and describe the price volatility characteristics of putable bonds.

We established earlier that the relation between price and yield for a straight coupon bond is negative. An increase in yield (discount rate) leads to a decrease in the value of a bond. The precise nature of this relationship for an option-free, 8 percent, 20-year bond is illustrated in Figure 2.

Figure 2: Price-Yield Curve for an Option-Free, 8%, 20-Year Bond



First, note that the price yield relationship is negatively sloped, so the price falls as the yield rises. Second, note that the relation follows a curve, not a straight line. Since the curve is convex (toward the origin) we say that an option-free bond has positive convexity. Because of convexity, the price of an option-free bond *increases more when yields fall than it decreases when yields rise.* In Figure 2 we have illustrated that, for an 8 percent, 20-year option-free bond, a 1 percent decrease in the YTM will increase the price to 110.67, a *10.67 percent increase* in price. A 1 percent increase in YTM will cause the bond value to decrease to 90.79, a *9.22 percent decrease* in value.

If the price-yield relation were a straight line, there would be no difference between the price increase and the price decline in response to equal decreases and increases in yields. Convexity is a good thing for a bond owner; for a given volatility of yields, price increases are larger than price decreases. The convexity property is often expressed by saying, "a bond's price falls at a decreasing rate as yields rise." For the price-yield relationship to be convex, the slope (rate of decrease) of the curve must be decreasing as we move from left to right (i.e., towards higher yields).

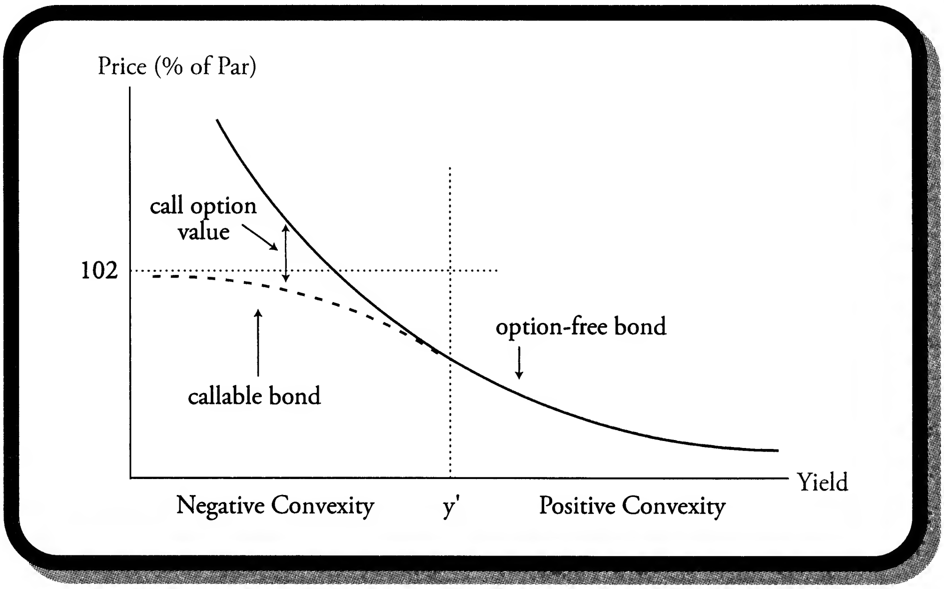
Note that the duration (interest rate sensitivity) of a bond at any yield is the (absolute value of) the slope of the price yield function at that yield. The convexity of the price yield relation for an option-free bond can help you remember a result presented earlier, that the duration of a bond is less at higher market yields. *(See Exam Flashback #4.)*

Callable Bonds, and Prepayable Securities, and Negative Convexity

With a **callable or prepayable debt**, the upside price appreciation in response to decreasing yields is limited (sometimes called price compression). Consider the case of a bond that is currently callable at 102. The fact that the issuer can call the bond at any time for $1,020 per $1,000 of face value puts an effective upper limit on the value of the bond. As Figure 3 illustrates, as yields fall and the price approaches $1,020, the price-yield curve rises more slowly than that of an identical but noncallable bond. When the price begins to *rise at a decreasing rate* in response to further decreases in yield, the price yield curve "bends over" to the left and exhibits **negative convexity**.

Thus, in Figure 3, so long as yields remain ***below level y'***, callable bonds will exhibit ***negative convexity;*** however, at yields ***above level y'***, those same callable bonds will exhibit ***positive convexity.*** In other words, at higher yields the value of the call options becomes very small so that a callable bond will act very much like a noncallable bond. It is only at lower yields that the callable bond will exhibit negative convexity.

Figure 3: Price-Yield Function of a Callable Bond



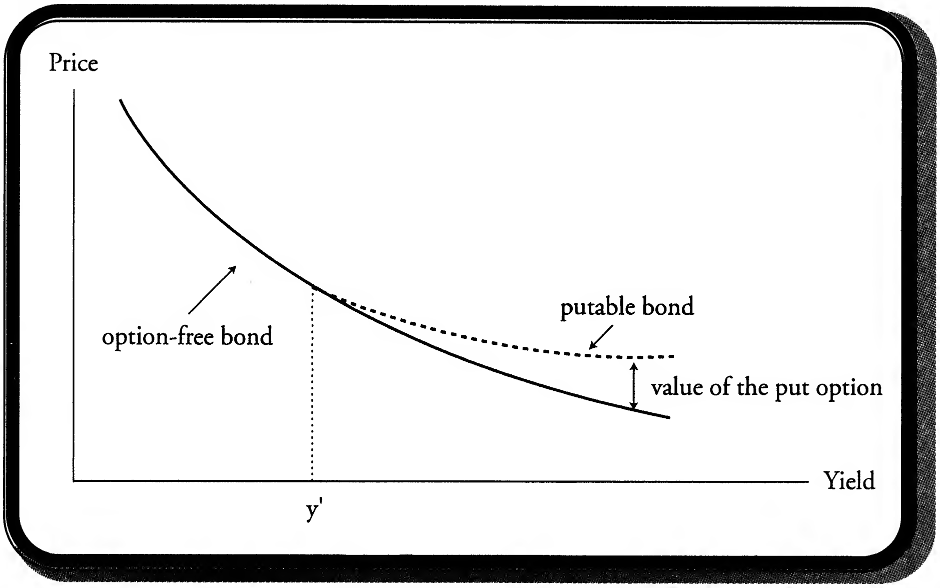
In terms of price sensitivity to interest rate changes, the slope of the price-yield curve at any particular yield tells the story. Note that as yields fall, the slope of the price-yield curve for the callable bond decreases, becoming almost zero (flat) at very low yields. This tells us how a call feature affects price sensitivity to changes in yield. At higher yields, the interest rate risk of a callable bond is very close or identical to that of a similar option-free bond. At lower yields, the price volatility of the callable bond will be much lower than that of an identical but noncallable bond.

The effect of a prepayment option is quite similar to that of a call; at low yields it will lead to negative convexity and reduce the price volatility (interest rate risk) of the security. Note that when yields are low and callable and prepayable securities exhibit less interest rate risk, reinvestment risk rises. At lower yields the probability of a call and the prepayment rate both rise, increasing the risk of having to reinvest principal repayments at the lower rates.

The Price Volatility Characteristics of Putable Bonds

The value of a put increases at higher yields and decreases at lower yields opposite to the value of a call option. Compared to an option-free bond, a **putable bond** will have *less* price volatility at higher yields. This comparison is illustrated in Figure 4.

Figure 4: Comparing the Price-Yield Curves for Option-Free and Putable Bonds



In Figure 4, the price of the putable bond falls more slowly in response to increases in yield above *y*' because the value of the embedded put rises at higher yields. The slope of the price-yield relation is flatter, indicating less price sensitivity to yield changes (lower duration) for the putable bond at higher yields. At yields below *y*', the value of the put is quite small, and a putable bond's price acts like that of an option-free bond in response to yield changes.

LOS 68.d: Compute the effective duration of a bond, given information about how the bond's price will increase and decrease for given changes in interest rates, and compute the approximate percentage price change for a bond, given the bond's effective duration and a specified change in yield.

In our introduction to the concept of duration, we described it as the ratio of the percentage change in price to change in yield. Now that we understand convexity, we know that the price change in response to rising rates is smaller than the price change in response to falling rates for option-free bonds. The formula we will use for calculating the **effective duration** of a bond uses the average of the price changes in response to equal increases and decreases in yield to account for this fact. If we have a callable bond that is trading in the area of negative convexity, the price increase is smaller than the price decrease, but using the average still makes sense.

The formula for calculating the effective duration of a bond is:

$$\text{effective duration} = \frac{(\text{bond price when yields fall} - \text{bond price when yields rise})}{2 \times (\text{initial price}) \times (\text{change in yield in decimal form})}$$

which we will sometimes write as $\text{duration} = \dfrac{V_- - V_+}{2V_0(\Delta y)}$

where:
V_- = bond value if the yield decreases by Δy
V_+ = bond value if the yield increases by Δy
V_0 = initial bond price
Δy = change yield used to get V_- and V_+, *expressed in decimal form*

Consider the following example of this calculation.

Example: Calculating effective duration

Consider a 20-year, semiannual-pay bond with an 8 percent coupon that is currently priced at $908.00 to yield 9 percent. If the yield declines by 50 basis points (to 8.5 percent), the price will increase to $952.30, and if the yield increases by 50 basis points (to 9.5 percent), the price will decline to $866.80. Based on these price and yield changes, **calculate** the effective duration of this bond.

Answer:

Let's approach this intuitively to gain a better understanding of the formula. We begin by computing the average of the percentage change in the bond's price for the yield increase and the percentage change in price for a yield decrease. We can calculate this as:

$$\text{average } \textit{percentage} \text{ price change} = \frac{(\$952.30 - \$866.80)}{2 \times \$908.00} = 0.0471\%, \text{ or } 4.71\%$$

The "2" in the denominator is to obtain the average price change, and the $908 in the denominator is to obtain this average change as a percentage of the current price.

To get the duration (to scale our result for a 1 percent change in yield), the final step is to divide this average percentage price change by the change in interest rates that caused it. In the example the yield change was 0.5 percent, which we need to write in decimal form as 0.005. Our estimate of the duration is:

$$\frac{0.0471}{0.005} = \frac{4.71\%}{0.50\%} = 9.42 = \text{duration}$$

Using the formula previously given, we have:

$$\text{effective duration} = \frac{(\$952.3 - \$866.8)}{2 \times \$908 \times 0.005} = 9.416$$

The interpretation of this result, as you should be convinced by now, is that a 1 percent change in yield produces an approximate change in the price of this bond of 9.42 percent. Note, however, that this estimate of duration was based on a change in yield of 0.5 percent and will perform best for yield changes close to this

magnitude. Had we used a yield change of 0.25 percent or 1 percent, we would have obtained a slightly different estimate of effective duration.

This is an important concept and you are required to learn the formula for the calculation. To further help you understand this formula and remember it, consider the following.

The price increase in response to a 0.5% decrease in rates was $\frac{\$44.30}{\$908} = 4.879\%$. The price decrease in response to a 0.5% increase in rates was $\frac{\$41.20}{\$908} = 4.537\%$. The average of the percentage price increase and the percentage price decrease is 4.71 percent. Since we used a 0.5 percent change in yield to get the price changes, we need to double this and get a 9.42 percent change in price for a 1 percent change in yield. The duration is 9.42.

Approximate Percentage Price Change for a Bond Based on Effective Duration

Multiply effective duration times the change in yield to get the magnitude of the price change and then change the sign to get the direction of the price change right (yield up, price down).

percentage change in bond price = –effective duration × change in yield in percent

Example: Using Effective Duration

What is the expected percentage price change for a bond with an effective duration of 9 in response to an increase in yield of 30 basis points?

Answer:

$-9 \times 0.3\% = -2.7\%$

We expect the bond's price to decrease by 2.7 percent in response to the yield change. If the bond were priced at \$980, the new price is 980 × (1 – 0.027) = \$953.54. Don't make this hard, it's not.
(See Exam Flashback #5 and #6.)

LOS 68.e: Distinguish among the alternative definitions of duration (modified, effective or option-adjusted, and Macaulay), explain why effective duration is the most appropriate measure of interest rate risk for bonds with embedded options, describe why duration is best interpreted as a measure of a bond's or portfolio's sensitivity to changes in interest rates, compute the duration of a portfolio, given the duration of the bonds comprising the portfolio, and discuss the limitations of portfolio duration.

The formula we used to calculate duration based on price changes in response to equal increases and decreases in YTM, $\text{duration} = \frac{V_{-} - V_{+}}{2V_0(\Delta y)}$, is the formula **for effective (option-adjusted) duration.** This is the preferred measure because it gives a good approximation of interest rate sensitivity for both option-free bonds and *bonds with embedded options.*

Macaulay duration is an estimate of a bond's interest rate sensitivity based on the time, in years, until promised cash flows will arrive. Since a 5-year zero coupon bond has only one cash flow five years from today, its Macaulay duration is 5. The change in value in response to a 1 percent change in yield for a 5-year zero coupon bond is approximately 5 percent. A 5-year coupon bond has some cash flows that arrive earlier than five years from today

(the coupons), so its Macaulay duration is less than 5. This is consistent with what we learned earlier: the higher the coupon, the less the price sensitivity (duration) of a bond.

Macaulay duration is the earliest measure of duration, and because it was based on the time, duration is often stated as years. Because Macaulay duration is based on the expected cash flows for an option-free bond, it is not an appropriate estimate of the price sensitivity of bonds with embedded options.

Modified duration is derived from Macaulay duration and offers a slight improvement over Macaulay duration in that it takes the current YTM into account. Like Macaulay duration, and for the same reasons, modified duration is not an appropriate measure of interest rate sensitivity for bonds with embedded options. For option-free bonds, however, effective duration (based on small changes in YTM) and modified duration will be very similar.

Professor's Note: The LOS here do not require that you calculate either Macaulay duration or modified duration, only effective duration. For your own understanding, however, note that the relation is:

$\textit{Modified duration} = \dfrac{\textit{Macaulay duration}}{\textit{1 + periodic market yield}}$. *This accounts for the fact we learned earlier that duration decreases as YTM increases. Graphically, the slope of the price-yield curve is less steep at higher yields.*

Effective Duration for Bonds With Embedded Options

As noted earlier, in comparing the various duration measures, both Macaulay and modified duration are calculated directly from the promised cash flows for a bond with no adjustment for the effect of any embedded options on cash flows. Effective duration is calculated from expected price changes in response to changes in yield that explicitly take into account a bond's option provisions (i.e., they are in the price-yield function used). *(See Exam Flashback #7.)*

Interpreting Duration

We can interpret duration in three different ways.

First, duration is the slope of the price-yield curve at the bond's current YTM. Mathematically, the slope of the price-yield curve is the first derivative of the price-yield curve with respect to yield.

A second interpretation of duration, as originally developed by Macaulay, is a weighted average of the time (in years) until each cash flow will be received. The weights are the proportions of the total bond value that each cash flow represents. The answer, again, comes in years.

A third interpretation of duration is the approximate percentage change in price for a 1 percent change in yield. This interpretation, price sensitivity in response to a change in yield, is the preferred, and most intuitive, interpretation of duration.

Professors Note: The fact that duration was originally calculated and expressed in years has been a source of confusion for many candidates and finance students. Practitioners regularly speak of "longer duration securities." This confusion is the reason for this part of the LOS. The most straightforward interpretation of duration is the one that we have used up to this point, "it is the approximate percentage change in a bond's price for a 1 percent change in YTM." I have seen duration expressed in years in CFA exam questions; just ignore the years and use the number. I have also seen questions asking whether duration becomes longer or shorter in response to a change; longer means higher or more interest rate sensitivity. A duration of 6.82 years means that for a 1 percent change in YTM, a bond's value will change approximately 6.82 percent. This is the best way to "interpret" duration.

Duration of a Portfolio

The concept of duration can also be applied to portfolios. In fact, one of the benefits of duration as a measure of interest rate risk is that the duration of a portfolio is simply the weighted average of the durations of the individual securities in the portfolio. Mathematically, the duration of a portfolio is:

$$\text{portfolio duration} = w_1 D_1 + w_2 D_2 + \ldots + W_N D_N$$

where:
w_i = market value of bond i divided by the market value of the portfolio
D_i = the duration of bond i
N = the number of bonds in the portfolio

Example: Calculating portfolio duration

Suppose you have a two-security portfolio containing Bonds A and B. The market value of Bond A is $6,000, and the market value of Bond B is $4,000. The duration of Bond A is 8.5, and the duration of Bond B is 4.0. **Calculate** the duration of the portfolio.

Answer:

First, find the weights of each bond. Since the market value of the portfolio is $10,000 = $6,000 + $4,000, the weight of each security is:

$$\text{weight in Bond A} = \frac{\$6{,}000}{\$10{,}000} = 60\%$$

$$\text{weight in Bond B} = \frac{\$4{,}000}{\$10{,}000} = 40\%$$

Using the formula for the duration of a portfolio, we get:

$$\text{portfolio duration} = (0.6 \times 8.5) + (0.4 \times 4.0) = 6.7$$

Limitations of Portfolio Duration

The limitations of portfolio duration as a measure of interest rate sensitivity stem from the fact that yields may not change equally on all the bonds in the portfolio. With a portfolio that includes bonds with different maturities, credit risks, and embedded options, there is no reason to suspect that the yields on individual bonds will change by equal amounts when the yield curve changes. As an example, a steepening of the yield curve can increase yields on long-term bonds and leave the yield on short-term bonds unchanged. It is for this reason that we say that duration is a good measure of the sensitivity of portfolio value to *parallel* changes in the yield curve.

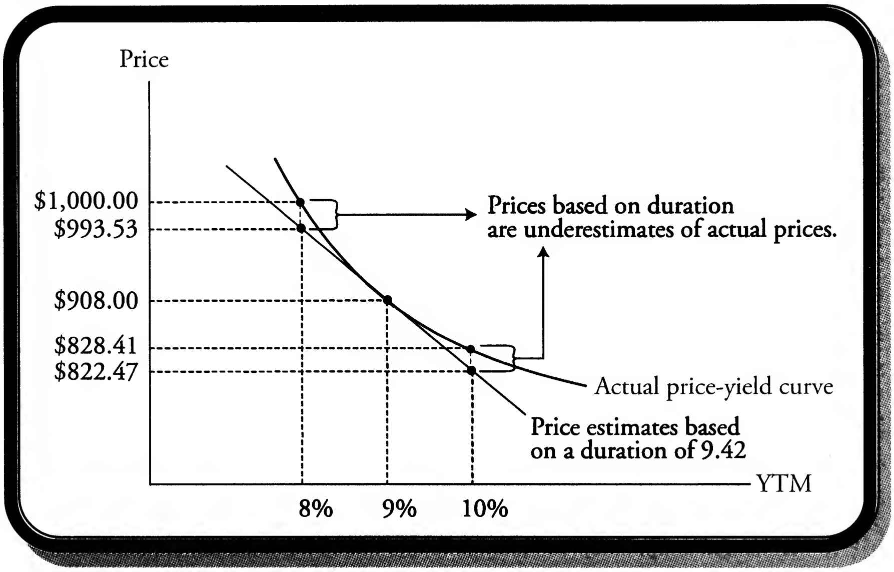
LOS 68.f: Discuss the convexity measure of a bond and estimate a bond's percentage price change, given the bond's duration and convexity and a specified change in interest rates.

Convexity is a measure of the curvature of the price-yield curve. The more curved the price-yield relation is, the greater the convexity. A straight line has a convexity of zero. If the price-yield 'curve' were, in fact, a straight line, the convexity would be zero. The reason we care about convexity is that the more curved the price-yield relation is, the worse our duration-based estimates of bond price changes in response to changes in yield are.

As an example, consider again an 8 percent, 20-year Treasury bond priced at $908 so that it has a yield to maturity of 9 percent. We previously calculated the effective duration of this bond as 9.42. Figure 5 illustrates

the differences between actual bond price changes and duration-based estimates of price changes at different yield levels.

Figure 5: Duration-Based Price Estimates vs. Actual Bond Prices



Based on a value of 9.42 for duration, we would estimate the new prices after 1% changes in yield (to 8% and to 10%) as 1.0942 × 908 = $993.53 and (1 – 0.0942) × 908 = $822.47, respectively. These price estimates are shown in Figure 5 along the straight line tangent to the actual price-yield curve.

The actual price of the 8 percent bond at a YTM of 8 percent is, of course, par value ($1,000). Based on a YTM of 10 percent, the actual price of the bond is $828.41, about $6 higher than our duration based estimate of $822.47. Note that price estimates based on duration are less than the actual prices for both a 1 percent increase and a 1 percent decrease in yield.

Figure 5 illustrates why convexity is important and why estimates of price changes based solely on duration are inaccurate. If the price-yield relation were a straight line (i.e., if convexity were zero), duration alone would provide good estimates of bond price changes for changes in yield of any magnitude. The greater the convexity, the greater the error in price estimates based solely on duration. A method of incorporating convexity into our estimates of bond price changes in response to yield changes is the subject of the next LOS. *(See Exam Flashback #8.)*

A Bond's Approximate Percentage Price Change Based on Duration and Convexity

By combining duration and convexity we can obtain a more accurate estimate of the percentage change in price of a bond, especially for relatively large changes in yield. The formula for estimating a bond's percentage price change based on its convexity and duration is:

percentage change in price = duration effect + convexity effect

$$= \left\{\left[-\text{duration} \times (\Delta y)\right] + \left[\text{convexity} \times (\Delta y)^2\right]\right\} \times 100$$

With Δy entered as a decimal, the "× 100" is necessary to get an answer in percent.

Example: Estimating price changes with duration and convexity

Consider an 8 percent Treasury bond with a current price of $908 and a YTM of 9 percent. **Calculate** the percentage change in price of both a 1 percent increase and a 1 percent decrease in YTM based on a duration of 9.42 and a convexity of 68.33.

Answer:

The duration effect, as we calculated earlier, is $9.42 \times 0.01 = 0.0942 = 9.42\%$. The convexity effect is $68.33 \times (0.01)^2 \times 100 = 0.00683 \times 100 = 0.683\%$. The total effect for a *decrease in yield of 1 percent* (from 9 percent to 8 percent) is: 9.42% + 0.683% = +10.103% and the estimate of the new price of the bond is 1.10103 × 908 = 999.74. This is much closer to the actual price of $1,000 than our estimate using only duration.

The total effect for an *increase in yield of 1 percent* (from 9 percent to 10 percent) is: –9.42% + 0.683% = –8.737% and the estimate of the bond price is (1 – 0.08737)(908) = $828.67. Again, this is much closer to the actual price ($828.40) than the estimate based solely on duration.

There are a few points worth noting here. First, the convexity adjustment is always positive when convexity is positive because $(\Delta y)^2$ is always positive. This goes along with the illustration in Figure 5, which shows that the duration-only based estimate of a bond's price change suffered from being an underestimate of the percentage increase in the bond price when yields fell, and an overestimate of the percentage decrease in the bond price when yields rose. Recall, that for a callable bond, convexity can be negative at low yields. When convexity is negative, the convexity adjustment to the duration-only based estimate of the percentage price change will be negative for both yield increases and yield decreases.

Professor's Note: Different dealers may calculate the convexity measure differently. Often the measure is calculated in a way that requires us to divide the measure by two in order to get the correct convexity adjustment. For exam purposes, the formula we've shown here is the one you need to know. However, you should also know that there can be some variation in how different dealers calculate convexity. (See Exam Flashbacks #9 and #10.)

LOS 68.g: Differentiate between modified convexity and effective convexity.

Effective convexity takes into account changes in cash flows due to embedded options, while modified convexity does not. The difference between modified convexity and effective convexity mirrors the difference between modified duration and effective duration. Recall that modified duration is calculated without any adjustment to a bond's cash flows for embedded options. Also recall that effective duration was appropriate for bonds with embedded options because the inputs (prices) were calculated under the assumption that the cash flows could vary at different yields because of the embedded options in the securities. Clearly, effective convexity is the appropriate measure to use for bonds with embedded options, since it is based on bond values that incorporate the effect of embedded options on the bond's cash flows.

LOS 68.h: Compute the price value of a basis point (PVBP), and explain its relationship to duration.

The price value of a basis point (PVBP) is the dollar change in the price/value of a bond or a portfolio when the yield changes by one basis point or 0.01 percent. We can calculate the PVBP directly for a bond by changing the YTM by one basis point and computing the change in value. As a practical matter, we can use duration to calculate the price value of a basis point as:

duration × 0.0001 × bond value = price value of a basis point

The following example demonstrates this calculation.

Example: Calculating the price value of a basis point

A bond has a market value of $100,000 and a duration of 9.42. What is the price value of a basis point?

Answer:

Using the duration formula, the percentage change in the bond's price for a change in yield of 0.01 percent is: $0.01\% \times 9.42 = 0.0942\%$. We can calculate 0.0942 percent of the original $100,000 portfolio value as $0.000942 \times 100{,}000 = \$94.20$. If the bond's yield increases (decreases) by 1 basis point, the portfolio value will fall (rise) by $94.20. $94.20 is the (duration-based) price value of a basis point for this bond.

We could also directly calculate the price value of a basis point for this bond by increasing the YTM by 0.01 percent (0.0001) and calculating the change in bond value. This would give us the PVBP for an increase in yield. This would be very close to our duration-based estimate because duration is a very good estimate of interest rate risk for small changes in yield. We can ignore the convexity adjustment here because it is of very small magnitude: $(\Delta y)^2 = (0.0001)^2 = 0.00000001$, which is pretty small indeed!

Key Concepts

1. The full valuation approach to measuring interest rate risk involves using a pricing model to value individual bonds and can be used to find the price impact of any scenario of interest rate/yield curve changes. Its advantages are its flexibility and precision.
2. The duration/convexity approach is based on summary measures of interest rate risk and, while simpler to use for a portfolio of bonds than the full valuation approach, is theoretically correct only for parallel shifts of the yield curve.
3. Callable bonds and prepayable securities will have less interest rate risk (lower duration) at lower yields and putable bonds will have less interest rate risk at higher yields, compared to option-free bonds.
4. Option-free bonds have a price-yield relationship that is curved (convex toward the origin) and are, therefore, said to exhibit positive convexity. Bond prices fall less rapidly in response to yield increases than they rise in response to lower yields.
5. Callable bonds exhibit negative convexity at low yield levels; bond prices rise less rapidly in response to yield decreases than they fall in response to yield increases.
6. Effective duration is calculated as the ratio of the average percentage price change for equal increases and decreases in yield to the change in yield, $\text{effective duration} = \dfrac{V_- - V_+}{2V_0(\Delta y)}$.
7. Percentage change in bond price = –duration × change in yield in percent.
8. Macaulay duration and modified duration are based on a bond's promised cash flows, while effective duration takes into account the effect of embedded options on a bond's cash flows.
9. Effective duration is appropriate for estimating price changes in bonds with embedded options in response to yield changes; Macaulay and modified duration are not.
10. The most intuitive interpretation of duration is as the percentage change in a bond's price for a 1 percent change in yield.
11. The duration of a portfolio of bonds is equal to a weighted average of the individual bond durations, where the weights are the proportions of total portfolio value in each bond position.
12. Portfolio duration is limited in that it gives the sensitivity of bond portfolio value to yield changes that are equal for all bonds in the portfolio, an unlikely scenario for most portfolios.
13. Because of convexity, the duration measure is a poor approximation of price sensitivity for yield changes that are not absolutely small. The convexity adjustment accounts for the curvature of the price-yield relationship.

14. Incorporating both duration and convexity we can estimate the percentage change in price in response to a change in yield of (Δy) as: $\left\{\left[-\text{duration} \times (\Delta y)\right] + \left[\text{convexity} \times (\Delta y)^2\right]\right\} \times 100$.
15. Effective convexity considers expected changes in cash flows that may occur for bonds with embedded options, while modified convexity does not.
16. PVBP measures the price impact, in dollars, of a 1 basis point change in yield on a bond or bond portfolio.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #102 from '98 sample exam.

Bond price volatility normally is:
A. lower for higher coupons.
B. lower for longer durations.
C. greater for shorter maturities.
D. none of the above.

Exam Flashback # 2

Source: Question #85 from '01–'03 sample exams

A fixed income manager wants to take advantage of a forecast decline in interest rates over the next several months. Which of the following combinations of maturity and coupon rate would *most likely* result in the largest increase in portfolio value?

	Maturity	Coupon Rate
A.	2015	10%
B.	2015	12%
C.	2030	10%
D.	2030	12%

Exam Flashback # 3

Source: Question #50 from 93 actual exam.

Philip Morris has issued bonds that pay interest semiannually with the following characteristics:

Coupon	Yield-to-Maturity	Maturity	Modified Duration
8%	8%	15 years	10 years

Identify the direction of change in modified duration if the coupon of the bond were 4%, not 8%.
A. Increase.
B. Decrease.
C. No change.
D. Cannot be determined with the information given.

Exam Flashback # 4

Source: Question #77 from '91 actual exam.

Which one of the following is an *incorrect* statement concerning duration?
A. The higher the yield-to-maturity, the greater the duration.
B. The higher the coupon, the shorter the duration.
C. The difference in duration is small between 2 bonds each maturing in more than 15 years.
D. For a zero coupon bond, duration is the same, or very close to, the bond's term-to-maturity.

Exam Flashback # 5

Source: Question #106 from '92, '96 actual exams, and '97, '98 sample exams

A 9-year bond has a yield-to-maturity of 10% and a modified duration of 6.54 years. If the market yield changes by 50 basis points, the bond's expected price change is:
A. 3.27%.
B. 3.66%.
C. 5.00%.
D. 6.54%.

Exam Flashback # 6

Source: Question #48 from '91 actual exam.

An 8%, 15-year bond has a yield-to-maturity of 10% and a modified duration of 8.05 years. If the market yield changes by 25 basis points, how much of the change in the bond's price will be due to duration?
A. 1.85%.
B. 2.01%.
C. 3.27%.
D. 6.44%.

Exam Flashback # 7

Source: Question #100 from '99, '00, '01, '02, '03 sample exams.

Interest rate sensitivity for bonds with embedded options is *most accurately* measured by:
A. Convexity.
B. Effective duration.
C. Modified duration.
D. Macaulay duration.

Exam Flashback # 8

Source: Question #66 from '90, '91 actual exams.

Positive convexity on a bond implies that:
A. the direction of change in yield is directly related to the change in price.
B. prices increase at a faster rate as yields drop, than they decrease as yields rise.
C. price changes are the same for both increases and decreases in yields.
D. prices increase and decrease at a faster rate than the change in yield.

Exam Flashback # 9

Source: Question #97 from '94 actual exam.

A 6 percent coupon bond with semiannual coupons has a *convexity* of 60, sells for 80 percent of par, and is priced at a yield to maturity (YTM) of 8 percent. If the YTM increases to 9.5 percent, the predicted contribution to the *percentage* change in price, due to convexity, would be:
A. 1.08%.
B. 1.35%.
C. 2.48%.
D. 7.35%.

Exam Flashback # 10

Source: Question #44 from '91 actual exam.

A certain agency bond has a duration of 8.73 years and a convexity of 61.33. This implies that:
A. if market yields increase significantly (e.g., rates increase by 250 basis points), the price of the bond will fall by less than the amount indicated by duration alone.
B. if market yields increase significantly, the price of the bond will fall by more than the amount indicated by duration alone.
C. if market yields decrease significantly (e.g., by 250 basis points), the price of the bond will increase by less than the amount indicated by the convexity measure alone.
D. if market yields decrease significantly, the price of the bond will increase by less than the amount indicated by duration alone.

Concept Checkers: Introduction to the Measurement of Interest Rate Risk

1. Why is the price/yield profile of a callable bond less convex than that of an otherwise identical option-free bond? The price:
   A. increase is capped from above at or near the call price as the required yield decreases.
   B. increase is capped from above at or near the call price as the required yield increases.
   C. decrease is limited from below at or near the call price as the required yield decreases.
   D. decrease is limited from below at or near the call price as the required yield increases.

2. You own $15 million face of the 4.65 percent semiannual-pay Portage Health Authority bonds. The bonds have exactly 17 years to maturity and are currently priced to yield 4.39 percent. Using the full valuation approach, the interest rate exposure (in percent of value) for this bond position given a 75 basis point increase in required yield is *closest* to:
   A. –9.104%.
   B. –9.031%.
   C. –8.344%.
   D. –8.283%.

3. You are estimating the interest rate risk of a 14 percent semiannual-pay coupon with 6 years to maturity. The bond is currently trading at par. Using a 25 basis point change in yield, the effective duration of the bond is *closest* to:
   A. 0.389.
   B. 0.397.
   C. 3.889.
   D. 3.970.

4. Suppose that the bond in Question 3 is callable at par today. Using a 25 basis point change in yield, the bond's effective duration assuming that its price cannot exceed 100 is *closest* to:
   A. 1.972.
   B. 1.998.
   C. 19.72.
   D. 19.98.

5. Suppose that you determine that the modified duration of a bond is 7.87. The percentage change in price using duration for a yield decrease of 110 basis points is *closest* to:
   A. –8.657%.
   B. –7.155%.
   C. +7.155%.
   D. +8.657%.

6. A bond has a convexity of 57.3. The convexity effect if the yield decreases by 110 basis points is *closest* to:
   A. –1.673%.
   B. –0.693%.
   C. +0.693%.
   D. +1.673%.

7. Assume you're looking at a bond that has an effective duration of 10.5 and a convexity of 97.3. Using both of these measures, the estimated percentage change in price for this bond, in response to a decline in yield of 200 basis points is *closest* to:
   A. 22.95%.
   B. 19.05%.
   C. 17.11%.
   D. 24.89%.

8. An analyst has determined that if market yields rise by 100 basis points, a certain high-grade corporate bond will have a convexity effect of 1.75 percent. Further, she's found that the total estimated percentage change in price for this bond should be –13.35 percent. Given this information, it follows that the bond's percentage change in price due to duration is:
   A. –15.10%.
   B. –11.60%.
   C. +15.10%.
   D. +16.85%.

9. The total price volatility of a typical noncallable bond can be found by:
   A. adding the bond's convexity effect to its effective duration.
   B. adding the bond's negative convexity to its modified duration.
   C. subtracting the bond's negative convexity from its positive convexity.
   D. subtracting the bond's modified duration from its effective duration, then add any positive convexity.

10. The current price of a $1,000 7-year 5.5 percent semiannual coupon bond is $1,029.23 The bond's PVBP is *closest* to:
    A. $5.93.
    B. $0.60.
    C. $0.05.
    D. $5.74.

11. The effect on a bond portfolio's value of a decrease in yield would be most accurately estimated by using:
    A. the price value of a basis point.
    B. the portfolio duration.
    C. the full valuation approach.
    D. both the portfolio's duration and convexity.

12. An analyst has noticed lately that the price of a particular bond has risen less when the yield falls by 0.1% than the price falls when rates increase by 0.1%. She could conclude that the bond:
    A. is an option-free bond.
    B. has an embedded put option.
    C. a zero-coupon bond.
    D. has negative convexity.

13. Which of the following measures is *lowest* for a currently callable bond?
    A. Macaulay duration.
    B. Effective duration.
    C. Modified duration.
    D. Cannot be determined.

Answers – Exam Flashbacks

1. **A** Volatility and duration carry the same meaning in this question. High coupon bonds exhibit lower volatility/duration, holding other bond characteristics constant.

2. **C** You are looking for the bond that has the longer maturity and the lower coupon. Hence, Bond C most likely has the highest volatility level.

3. **A** As the coupon declines, the interest rate sensitivity of the bond (duration) increases.

4. **A** Because of convexity, duration is less for higher yields to maturity.

5. **A** $\Delta P/P = (-)(MD)(\Delta y) = (-)(6.54)(\pm 0.005) = \pm 0.0327$ or 3.27%

6. **B** $\Delta P/P = (-)(MD)(\Delta y) = (-)(8.05)(\pm 0.0025) = \pm 0.0201$ or 2.01%

7. **B** Effective duration accounts for changes in the curvature of the price-yield function that occur for bonds with embedded options. As yields fall, prices rise at a *decreasing rate* for bonds with embedded call options. Neither modified duration nor Macaulay duration would capture this impact. Convexity measures the rate of change in duration and is not a primary measure of interest rate sensitivity.

8. **B** Convexity is a measure of the rate of change in duration. Recall that duration is a measure of the slope of the price-yield function. Hence, as rates fall, the slope rises at an increasing rate and as yields rise, the slope flattens out. Said differently, as rates fall, prices rise at an increasing rate and as rates rise, prices fall at a decreasing rate.

9. **B** $(C)(\Delta y)^2 = (60)(0.015)^2 = 0.0135 = 1.35\%$. Note that the decimal change in interest rates was used in this formula.

10. **A** Duration *overestimates* the decline in price due to an increase in interest rates. This is due to the fact that the price-yield function lies everywhere above the linear duration approximation.

Answers – Concept Checkers: Introduction to the Measurement of Interest Rate Risk

1. **A** As the required yield decreases on a callable bond, the rate of increase in the price of the bond begins to slow down and eventually level off as it approaches the call price, a characteristic known as "negative convexity."

2. **C** We need to compare the value of the bond today to the value if the YTM increases by 0.75%

   Price today = 103.092

   $$N = 34;\ PMT = \frac{4.65}{2} = 2.325;\ FV = 100;\ I/Y = \frac{4.39}{2} = 2.195\%;\ CPT \rightarrow PV = -103.092$$

   Price after 75 basis point increase in interest rates = 94.490

   $$N = 34;\ PMT = \frac{4.65}{2} = 2.325;\ FV = 100;\ I/Y = \frac{5.14}{2} = 2.57\%;\ CPT \rightarrow PV = -94.490$$

   $$\text{Interest rate exposure} = \frac{94.490 - 103.092}{103.092} = -8.344\%$$

3. **D** $V_- = 100.999$

$$N = 12;\ PMT = \frac{14.00}{2} = 7.00;\ FV = 100;\ I/Y = \frac{13.75}{2} = 6.875\%;\ CPT \rightarrow PV = -100.999$$

$V_+ = 99.014$

$$N = 12;\ PMT = \frac{14.00}{2} = 7.00;\ FV = 100;\ I/Y = \frac{14.25}{2} = 7.125\%;\ CPT \rightarrow PV = -99.014$$

$V_0 = 100.000$

$\Delta y = 0.0025$

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)} = \frac{100.999 - 99.014}{2(100)0.0025} = 3.970$$

4. **A** $V_- = 100.000$

$V_+ = 99.014$

$V_0 = 100.000$

$\Delta y = 0.0025$

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)} = \frac{100.000 - 99.014}{2(100)0.0025} = 1.972$$

5. **D** $\text{Est.}[\Delta V\_\%] = -7.87 \times (-1.10\%) = 8.657\%$

6. **C** convexity effect = convexity × $(\Delta y)^2$ $= \left[57.3(0.011)^2\right] \times 100 = 0.693\%$

7. **D** Total estimated price change = (duration effect + convexity effect)

$$\left\{\left[-10.5 \times (-0.02)\right] + \left[97.3 \times (-0.02)^2\right]\right\} \times 100 = 21.0\% + 3.89\% = 24.89\%$$

8. **A** Total percentage change in price = duration effect + convexity effect. Thus:

$$-13.35 = \text{duration effect} + 1.75 \Rightarrow \text{duration effect} = -15.10\%$$

(Note the duration effect must be negative because yields are rising.)

9. **A** Total percentage change in price = duration effect + convexity effect. Thus:

   Total percentage change in price = effective duration + convexity effect

   (Note that since this is a noncallable bond, you can use either effective or modified duration in the above equation.)

10. **B** PVBP = initial price – price if yield is changed by 1 bp. First, we need to calculate the yield so that we can calculate the price of the bond with a 1 basis point change in yield. Using a financial calculator: PV = –1,029.23; FV = 1,000; PMT = 27.5 = (0.055 × 1,000) / 2; N =14 = 2 × 7 years; CPT → I/Y = 2.49998, multiplied by 2 = 4.99995, or 5.00%. Next, compute the price of the bond at a yield of 5.00% + 0.01%, or 5.01%. Using the calculator: FV = 1,000; PMT = 27.5; N = 14; I/Y = 2.505 (5.01 / 2); CPT → PV = $1,028.63. Finally, PVBP = $1,029.23 – $1,028.63 = $0.60.

11. **C** The full valuation approach is the most complex method, but also the most accurate.

12. **D** A bond with negative convexity will rise less in price in response to a decrease in yield than it will fall in response to an equal-sized increase in rates.

13. **B** The interest rate sensitivity of a bond with an embedded call option will be less than that of an option-free bond. Effective duration takes the effect of the call option into account and will, therefore, be less than Macaulay or modified duration.

The following is a review of the Derivative Investments principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

Derivative Markets and Instruments

Study Session 16

Exam Focus

This topic review contains introductory material for the upcoming reviews of specific types of derivatives. Derivatives-specific definitions and terminology are presented along with information about derivatives markets. Upon completion of this review, candidates should be familiar with the basic concepts that underlie derivatives securities and the general arbitrage framework. There is little contained in this review that will not be elaborated upon in the five reviews that follow.

LOS 69.a: Define a derivative and differentiate between exchange-traded and over-the-counter derivatives.

A **derivative** is a security that *derives* its value from the value or return of another asset or security.

A physical exchange exists for many options contracts and futures contracts. **Exchange-traded derivatives** are standardized and backed by a clearinghouse.

Forwards and *swaps* are custom instruments and are traded/created by dealers in a market with no central location. A dealer market with no central location is referred to as an **over-the-counter** market. They are largely unregulated markets and each contract is with a counterparty, which may expose the owner of a derivative to default risk (when the counterparty does not honor their commitment).

Some *options* trade in the over-the-counter market, notably bond options.

LOS 69.b: Define a forward commitment, identify the types of forward commitments, and describe the basic characteristics of forward contracts, futures contracts, and swaps.

A **forward commitment** is a legally binding promise to perform some action in the future. Forward commitments include forward contracts, futures contracts, and swaps.

Forward contracts and futures contracts can be written on equities, indexes, bonds, physical assets, or interest rates.

In a **forward contract** one party agrees to buy, and the counterparty to sell, a physical asset or a security at a specific price on a specific date in the future. If the future price of the asset increases, the buyer (at the older, lower price) has a gain, and the seller a loss.

A **futures contract** is a forward contract that is standardized and exchange-traded. The main differences with forwards are that futures are traded in an active secondary market, are regulated, backed by the clearinghouse, and require a daily settlement of gains and losses.

A **swap** is a series of forward contracts. In the simplest swap, one party agrees to pay the short-term (floating) rate of interest on some principal amount, and the counterparty agrees to pay a certain (fixed) rate of interest in return. Swaps of different currencies and equity returns are also common.

LOS 69.c: Define a contingent claim and identify the types of contingent claims.

A **contingent claim** is a claim (to a payoff) that depends on a particular event. Options are contingent claims that depend on a stock price at some future date. Options are often included in bonds. The option to convert to common stock and the firm's option to call the bond are both contingent claims that only have value if certain events occur. While forwards, futures, and swaps have payments that are made based on a price or rate outcome whether the movement is up or down, contingent claims only require a payment if a certain threshold price is broken (i.e., if the price is above X or the rate is below Y). It takes two options to make a future or forward.

LOS 69.d: Describe the basic characteristics of options, and distinguish between an option to buy (call) and an option to sell (put).

An option to buy an asset at a particular price is termed a **call option**. The seller of the option has an *obligation* to sell the asset at the agreed-upon price, if the call buyer chooses to exercise the right to buy the asset.

An option to sell an asset at a particular price is termed a **put option**. The seller of the option has an *obligation* to purchase the asset at the agreed-upon price, if the put buyer chooses to exercise the right to sell the asset.

Professor's Note: To remember these terms, note that the owner of a call can "call the asset in" (i.e., buy it); the owner of a put has the right to "put the asset to" the writer of the put.

LOS 69.e: Discuss the purposes and criticisms of derivative markets.

The **criticism of derivatives** is that they are "too risky," especially to investors with limited knowledge of sometimes complex instruments. Because of the high leverage involved in derivatives payoffs, they are sometimes likened to gambling.

The **benefits of derivatives** markets are that they:

- Provide price information.
- Allow risk to be managed and shifted among market participants.
- Reduce transactions costs.

LOS 69.f: Explain the concept of arbitrage and the role it plays in determining prices and in promoting market efficiency.

Arbitrage is an important concept in valuing (pricing) derivative securities. In its purest sense, arbitrage is riskless. If a return greater than the risk-free rate can be earned by holding a portfolio of assets that produces a certain (riskless) return, then an arbitrage opportunity exists.

Arbitrage opportunities arise when assets are mispriced. Trading by arbitrageurs will continue until they affect supply and demand enough to bring asset prices to efficient (no-arbitrage) levels.

There are two arbitrage arguments that are particularly useful in the study and use of derivatives.

The first is based on the "law of one price." Two securities or portfolios that have identical cash flows in the future, regardless of future events, should have the same price. If A and B have the identical future payoffs, and A is priced lower than B, buy A and sell B. You have an immediate profit, and the payoff on A will satisfy your (future) liability of being short B.

The second type of arbitrage is used where two securities with uncertain returns can be combined in a portfolio that will have a certain payoff. If a portfolio consisting of A and B has a certain payoff, the portfolio should yield the risk-free rate. If this no-arbitrage condition is violated in that the certain return of A and B together is higher than the risk-free rate, an arbitrage opportunity exists. An arbitrageur could borrow at the risk-free rate, buy the

A + B portfolio, and earn arbitrage profits when the certain payoff occurs. The payoff will be more than is required to pay back the loan at the risk-free rate.

Professor's Note: We will discuss arbitrage in our review of options.

KEY CONCEPTS

1. A derivative has a value that is "derived" from the value of another asset or security.
2. Exchange-traded derivatives, notably options and futures, are traded in centralized locations, and are standardized, regulated, and without the risk of default.
3. Forward and swap contracts are traded in over-the-counter markets. These markets consist of dealers who offer customized contracts. There is very limited liquidity (secondary trading), and default risk is a concern.
4. A forward commitment is a binding promise to buy or sell an asset or make a payment in the future. Forward contracts are common with physical assets, interest rates, bonds, and equities.
5. Forward contracts obligate one party to buy, and another to sell, a specific asset at a predetermined price on a certain date in the future.
6. Swaps contracts are a series of forward contracts, often on interest rates but also on currencies and equity returns.
7. Futures contracts are forward contracts that are exchange-traded, quite liquid, and require daily settlement of any gains or losses.
8. A contingent claim is an asset that has value only if some future event takes place (e.g., asset price is greater than a specified price).
9. A call option gives the holder the right, but not the obligation, to buy an asset at a predetermined price at some time in the future.
10. A put option gives the holder the right, but not the obligation, to sell an asset at a predetermined price at some time in the future.
11. Derivative markets are criticized for their risky nature; however, many market participants use derivatives to manage and reduce risk.
12. Derivatives play an important role in promoting efficient market prices and lowering transaction costs.
13. Riskless arbitrage involves earning over the risk-free rate with no risk or earning an immediate gain with no future liability.

CONCEPT CHECKERS: DERIVATIVE MARKETS AND INSTRUMENTS

1. Which of the following *most accurately* describes a derivative security? A derivative:
   A. has no risk.
   B. always increases risk.
   C. has no expiration date.
   D. has a payoff based on another asset.

2. Which of the following statements about exchange-traded derivatives is **FALSE**?
   A. They are liquid.
   B. They provide price information.
   C. They are standardized contracts.
   D. They carry significant default risk.

3. A customized agreement to purchase a certain T-bond next Thursday for $1,000 is:
   A. a swap.
   B. an option.
   C. a futures contract.
   D. a forward commitment.

4. A futures contract is all of the following **EXCEPT**:
   A. liquid.
   B. exchange-traded.
   C. a contingent claim.
   D. adjusted for profits and losses daily.

5. A swap is:
   A. highly regulated.
   B. a series of options contracts.
   C. a series of forward contracts.
   D. the exchange of one asset for another.

6. A call option gives the holder:
   A. the right to sell at a specific price.
   B. the right to buy at a specific price.
   C. an obligation to buy at a certain price.
   D. an obligation to sell at a certain price.

7. Arbitrage prevents:
   A. risk management.
   B. market efficiency.
   C. profit higher than the risk-free rate of return.
   D. two assets with identical payoffs from selling at different prices.

8. Derivatives can provide or improve all of the following **EXCEPT**:
   A. liquidity.
   B. risk reduction.
   C. price information.
   D. inflation reduction.

ANSWERS – CONCEPT CHECKERS: DERIVATIVE MARKETS AND INSTRUMENTS

1. **D** A derivative's value is "derived" from another asset.

2. **D** Exchange-traded derivatives have relatively low default risk because the clearinghouse stands between the counterparties involved in most contracts.

3. **D** This non-standardized type of contract is a forward commitment.

4. **C** A contingent claim has payoffs that depend on some future event (e.g., an option).

5. **C** A swap is an agreement to buy or sell an underlying asset periodically over the life of the swap contract. It is equivalent to a series of forward contracts.

6. **B** A call gives the owner the right to call an asset away (buy it) from the seller.

7. **D** Arbitrage forces two assets with the same expected future value to sell for the same current price. If this were not the case, you could simultaneously buy the cheaper asset and sell the more expensive one for a guaranteed riskless profit.

8. **D** Inflation is a monetary phenomenon, unaffected by derivatives.

The following is a review of the Derivative Investments principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

FORWARD MARKETS AND CONTRACTS

Study Session 16

EXAM FOCUS

This topic review introduces forward contracts in general and covers the characteristics of forward contracts on various financial securities as well as interest rates. It is not easy material, and you should take the time to learn it well. This material on forward contracts provides a good basis for futures contracts and many of the characteristics of both types of contracts are the same. Take the time to understand the intuition behind the valuation of forward rate agreements so that you can value them without the formula; that way you don't have to rely on your memory for the formula.

WARM-UP: FORWARD CONTRACTS

A *forward contract* is a bilateral contract that obligates one party to buy and the other to sell a specific quantity of an asset, at a set price, on a specific date in the future. Typically, neither party to the contract pays anything to get into the contract. If the expected future price of the asset increases over the life of the contract, the right to buy at the contract price will have positive value, and the obligation to sell will have an equal negative value. If the future price of the asset falls below the contract price, the result is opposite and the right to sell (at an above-market price) will have the positive value. The parties may enter into the contract as a speculation on the future price. More often, a party seeks to enter into a forward contract to hedge a risk they already have. The forward contract is used to eliminate uncertainty about the future price of an asset they plan to buy or sell at a later date. Forward contracts on physical assets, such as agricultural products, have existed for centuries. The Level 1 CFA curriculum, however, focuses on their (more recent) use for financial assets, such as T-bills, bonds, equities, and foreign currencies.

LOS 70.a: Discuss the differences between the positions held by the long and short parties to a forward contract in terms of delivery/settlement and default risk.

The party to the forward contract that agrees to buy the financial or physical asset has a **long forward position** and is called the *long*. The party to the forward contract that agrees to sell or deliver the asset has a **short forward position** and is called the *short*.

We will illustrate the mechanics of the basic forward contract through an example based on the purchase and sale of a Treasury bill. Note that while forward and futures contracts on T-bills are usually quoted in terms of a discount percentage from face value, we will use dollar prices to make the example easy to follow. Actual pricing conventions and calculations are among the contract characteristics covered later in this review.

Consider a contract under which party A agrees to buy a \$1,000 face value, 90-day Treasury bill from party B 30 days from now at a price of \$990. Party A is the long and party B is the short. Both parties have removed uncertainty about the price they will pay/receive for the T-bill at the future date. If 30 days from now T-bills are trading at \$992, the short must deliver the T-bill to the long in exchange for a \$990 payment. If T-bills are trading at \$988 on the future date, the long must purchase the T-bill from the short for \$990, the contract price.

Each party to a forward contract is exposed to *default risk*, the probability that the other party (the counterparty) will not perform as promised. It is unusual for any cash to actually be exchanged at the inception of a forward

contract, unlike futures contracts in which each party posts an initial deposit (margin) as a guarantee of performance.

At any point in time, including the settlement date, only one party to the forward contract will "owe" money, meaning that side of the contract has a negative value. The other side of the contract will have a positive value of an equal amount. Following the example, if the T-bill price is $992 at the (future) settlement date and the short does not deliver the T-bill for $990 as promised, the short has defaulted.

LOS 70.b: Describe the procedures for settling a forward contract at expiration, and discuss how a party to a forward contract can terminate a position prior to expiration as well as how credit risk is affected by the way in which a position is terminated.

The previous example was for a **deliverable forward contract**. The short contracted to deliver the actual instrument, in this case a $1,000 face value, 90-day T-bill.

This is one procedure for settling a forward contract at the *settlement date* or expiration date specified in the contract.

An alternative settlement method is **cash settlement**. Under this method, the party that has a position with negative value is obligated to pay that amount to the other party. In the previous example, if the price of the T-bill were $992 on the expiration date, the short would satisfy the contract by paying $2 to the long. Ignoring transactions costs, this method yields the same result as asset delivery. If the short had the T-bill, it could be sold in the market for $992. The short's net proceeds, however, would be $990 after subtracting the $2 payment to the long. If the T-bill price at the settlement date were $988, the long would make a $2 payment to the short. Purchasing a T-bill at the market price of $988, together with this $2 payment, would make the total cost $990, just as it would be if it were a deliverable contract.

On the expiration (or settlement) date of the contract, the long receives a payment if the price of the asset is above the agreed-upon (forward) price; the short receives a payment if the price of the asset is below the contract price.

Terminating a Position Prior to Expiration

A party to a forward contract can **terminate the position** prior to expiration by entering into an opposite forward contract with an expiration date equal to the time remaining on the original contract.

Recall our example and assume that 10 days after inception (it was originally a 30-day contract) the 20-day forward price of a $1,000 face value, 90-day T-bill is $992. The short, expecting the price to be even higher by the delivery date, wishes to terminate the contract. Since the short is obligated to sell the T-bill 20 days in the future, he can effectively exit the contract by entering into a new (20-day) forward contract to buy an identical T-bill (a long position) at the current forward price of $992.

The position of the original short now is two-fold, an obligation to sell a T-bill in 20 days for $990 (under the original contract) and an obligation to purchase an identical T-bill in 20 days for $992. He has "locked in" a $2 loss, but has effectively exited the contract since the amount owed at settlement is $2, regardless of the market price of the T-bill at the settlement date. No matter what the price of a 90-day T-bill is 20 days from now, he has the contractual right and obligation to buy one at $992 and to sell one at $990.

However, if the short's new forward contract is with a different party than the first forward contract, some **credit risk** remains. If the price of the T-bill at the expiration date is above $992, and the counterparty to the second forward contract fails to perform, the short's losses could exceed $2.

An alternative is to enter into the second (offsetting) contract with the same party as the original contract. This would avoid **credit risk** since the short could make a $2 payment to the counterparty at contract expiration, the

amount of his net exposure. In fact, if the original counterparty were willing to take the short position in the second (20-day) contract at the $992 price, a payment of the present value of the $2 (discounted for the 20 days until the settlement date) would be an equivalent transaction. The original counterparty would be willing to allow termination of the original contract for an immediate payment of that amount.

If the original counterparty requires a payment larger than the present value of $2 to exit the contract, the short must weight this additional cost to exit the contract against the default risk he bears by entering into the offsetting contract with a different counterparty at a forward price of $992.

LOS 70.c: Differentiate between a dealer and an end user of a forward contract.

The **end user of a forward contract** is typically a corporation, government unit, or nonprofit institution that has existing risk they wish to avoid by locking in the future price of an asset. A U.S. corporation that has an obligation to make a payment in Euros 60 days from now can eliminate its exchange rate risk by entering into a forward contract to purchase the required amount of Euros for a certain dollar-denominated payment with a settlement date 60 days in the future.

Dealers are often banks but can also be nonbank financial institutions such as Merrill Lynch. Ideally, dealers will balance their overall long positions with their overall short positions by entering forward contracts with end users who have opposite existing risk exposures. A dealer's quote desk will quote a buying price (at which they will assume a long position) and a slightly higher selling price (at which they will assume a short position). The bid/ask spread between the two is the dealer's compensation for administrative costs as well as bearing default risk and any asset price risk from unbalanced (unhedged) positions. Dealers will also enter into contracts with other dealers to hedge a net long or net short position.

LOS 70.d: Describe the characteristics of equity forward contracts.

Equity forward contracts where the underlying asset is a single stock, a portfolio of stocks, or a stock index, work in much the same manner as other forward contracts. An investor who wishes to sell 10,000 shares of IBM stock 90 days from now and wishes to avoid the uncertainty about the stock price on that date, could do so by taking a short position in a forward contract covering 10,000 IBM shares. (We will leave the motivation for this and the pricing of such a contract aside for now.)

A dealer might quote a price of $100 per share, agreeing to pay $1 million for the 10,000 shares 90 days from now. The contract may be deliverable or settled in cash as described above. The stock seller has locked in the selling price of the shares and will get no more if the price (in 90 days) is actually higher, and will get no less if the price actually lower.

A portfolio manager who wishes to sell a portfolio of several stocks 60 days from now can similarly request a quote, giving the dealer the company names and the number of shares of each stock in the portfolio. The only difference between this type of forward contract and several forward contracts each covering a single stock, is that the pricing would be better (a higher total price) for the portfolio because overall administration/origination costs would be less for the portfolio forward contract.

A forward contract on a stock index is similar except that the contract will be based on a notional amount and will very likely be a cash-settlement contract.

Example: Equity index forward contracts

A portfolio manager desires to generate $10 million 100 days from now from a portfolio that is quite similar in composition to the S&P 100 index. She requests a quote on a short position in a 100-day forward contract based on the index with a notional amount of $10 million and gets a quote of 525.2. If the index level at the settlement date is 535.7, **calculate** the amount the manager will pay or receive to settle the contract.

Answer:

The actual index level is 2 percent *above* the contract price, or:

$535.7 / 525.2 - 1 = 0.02 = 2\%$

As the short party, the portfolio manager must pay 2 percent of the $10 million notional amount, $200,000, to the long.

Alternatively, if the index were one percent below the contract level, the portfolio manager would receive a payment from the long of $100,000, which would approximately offset any decrease in the portfolio value.

Dividends are usually not included in equity forward contracts, as the uncertainty about dividend amounts and payment dates is small compared to the uncertainty about future equity prices. Since forward contracts are custom instruments, the parties could specify a total return value (including dividends) rather than simply the index value. This would effectively remove dividend uncertainty as well.

LOS 70.e: Describe the characteristics of forward contracts on zero-coupon and coupon bonds.

Forward contracts on short-term, zero-coupon bonds (T-bills in the U.S.) and coupon interest-paying bonds are quite similar to those on equities. However, while equities do not have a maturity date, bonds do, and the forward contract must settle before the bond matures.

As we noted earlier, T-bill prices are often quoted as a percentage discount from face value. The percentage discount for T-bills is annualized so that a 90-day T-bill quoted at a 4 percent discount will be priced at a $(90 / 360) \times 4$ percent = 1 percent discount from face value. This is equivalent to a price quote of $(1 - 0.01) \times \$1{,}000 = \$990$ per $1,000 of face value.

Example: Bond forwards

A forward contract covering a $10 million face value of T-bills that will have 100 days to maturity at contract settlement is priced at 1.96 on a discount yield basis. **Compute** the dollar amount the long must pay at settlement for the T-bills.

Answer

The 1.96 percent annualized discount must be "unannualized" based on the 100 days to maturity.

$0.0196 \times (100/360) = 0.005444$ is the actual discount.

The dollar settlement price is $(1 - 0.005444) \times \$10$ million = $9,945,560.

Please note that when market interest rates increase, discounts increase, and T-bill prices fall. A long, who is obligated to purchase the bonds, will have losses on the forward contract when interest rates rise, and gains on the contract when interest rates fall. The outcomes for the short will be opposite.

The price specified in forward contracts on coupon-bearing bonds is typically stated as a yield to maturity as of the settlement date, exclusive of accrued interest. If the contract is on bonds with the possibility of default, there must be provisions in the contract to define default and specify the obligations of the parties in the event of default. Special provisions must also be included if the bonds have embedded options such as call features or conversion features. Forward contracts can be constructed covering individual bonds or portfolios of bonds.

LOS 70.f: Explain the characteristics of the Eurodollar time deposit market, define LIBOR and Euribor, and describe the characteristics of forward rate agreements (FRAs).

Eurodollar deposit is the term for deposits in large banks outside the United States denominated in U.S. dollars. The lending rate on dollar-denominated loans between banks is called the London Interbank Offered Rate (LIBOR). It is quoted as an annualized rate based on a 360-day year. In contrast to T-bill discount yields, LIBOR is an add-on rate, like a yield quote on a short-term certificate of deposit. LIBOR is used as a reference rate for floating rate U.S. dollar-denominated loans worldwide.

Example: LIBOR-based loans

Compute the amount that must be repaid on a $1 million loan for 30 days if 30-day LIBOR is quoted at 6 percent.

Answer:

The add-on interest is calculated as $1 million × 0.06 × (30 / 360) = $5,000. The borrower would repay $1,000,000 + $5,000 = $1,005,000 at the end of 30 days.

LIBOR is published daily by the British Banker's Association and is compiled from quotes from a number of large banks; some are large multinational banks based in other countries that have London offices.

There is also an equivalent Euro lending rate called Euribor, or Europe Interbank Offered Rate. Euribor, established in Frankfurt, is published by the European Central Bank.

The floating rates are for various periods and are quoted as such. For example, the terminology is 30-day LIBOR (or Euribor), 90-day LIBOR, and 180-day LIBOR, depending on the term of the loan. For longer-term floating rate loans, the interest rate is reset periodically based on the then-current LIBOR for the relevant period.

Characteristics of Forward Rate Agreements (FRAs)

A **forward rate agreement** (FRA) can be viewed as a forward contract to borrow/lend money at a certain rate at some future date. In practice, these contracts settle in cash, but no actual loan is made at the settlement date. This means that the creditworthiness of the parties to the contract need not be considered in the forward interest rate, so an essentially riskless rate, such as LIBOR, can be specified in the contract. (The parties to the contract may still be exposed to default risk on the amount owed at settlement.)

The long position in an FRA is the party that would borrow the money (long the loan with the contract price being the interest rate on the loan). If the floating rate at contract expiration (LIBOR or Euribor) is above the rate specified in the forward agreement, the long position in the contract can be viewed as the right to borrow at below market rates and the long will receive a payment. If the reference rate at the expiration date is below the contract rate, the short will receive a cash payment from the long. (The right to lend at rates *higher than* market rates would have a positive value.)

LOS 70.g: Calculate and interpret the payment at expiration of an FRA, explain each of the component terms, and describe the characteristics of currency forward contracts.

To calculate the cash payment at settlement for a forward rate agreement, we need to calculate the value as of the settlement date of making a loan at a rate that is either above or below the market rate. Since the interest savings would come at the end of the "loan" period, the cash payment at settlement of the forward is the present value of the interest "savings." We need to calculate the discounted value at the settlement date of the interest savings or excess interest at the end of the loan period. An example will illustrate the calculation of the payment at expiration and some terminology of FRAs.

Example: FRAs

Consider an FRA that:

- Expires/settles in 30 days.
- Is based on a notional principal amount of $1 million.
- Is based on 90-day LIBOR.
- Specifies a forward rate of 5%.

Assume that the actual 90-day LIBOR 30-days from now (at expiration) is 6 percent. **Compute** the cash settlement payment at expiration, and **identify** which party makes the payment.

Answer:

If the long could borrow at the contract rate of 5 percent, rather than the market rate of 6 percent, the interest saved on a 90-day $1 million loan would be:

$$(0.06 - 0.05)(90 / 360) \times 1 \text{ million} = 0.0025 \times 1 \text{ million} = \$2{,}500$$

The $2,500 in interest savings would not come until the end of the 90-day loan period. The value at settlement is the present value of these savings. The correct discount rate to use is the actual rate at settlement, 6 percent, not the contract rate of 5 percent.

The payment at settlement from the short to the long is:

$$\frac{2{,}500}{1 + (0.06) \times (90/360)} = \$2{,}463.05$$

In doing the calculation of the settlement payment, remember that the term of the FRA and the term of the underlying "loan" need not be the same and are *not* interchangeable. While the settlement date can be any future date, in practice it is usually some multiple of 30 days. The specific market rate on which we calculate the value of the contract will typically be similar, 30-day, 60-day, 90-day, or 180-day LIBOR. If we describe an FRA as a 60-day FRA on 90-day LIBOR, settlement or expiration is 60 days from now and the payment at settlement is based on 90-day LIBOR 60 days from now. Such an FRA could be quoted in (30-day) months, and would be described as a 2-by-5 FRA (or 2 × 5 FRA). The 2 refers to the number of months until contract expiration and the 5 refers to the total time until the end of the interest rate period (2 + 3 = 5).

The general formula for the payment to the long at settlement is:

$$(\text{notional principal})\frac{(\text{floating} - \text{forward})\left(\frac{\text{days}}{360}\right)}{1 + (\text{floating})\left(\frac{\text{days}}{360}\right)}$$

$$\begin{aligned}&\text{where:}\\&\text{days} = \text{number of days in the loan term}\end{aligned}$$

The numerator is the "interest savings" in percent, and the denominator is the discount factor.

Note that if the *floating* rate underlying the agreement turns out to be below the *forward* rate specified in the contract, the numerator in the formula is negative and the short receives a payment from the long.

FRAs for non-standard periods (e.g., a 45-day FRA on 132-day LIBOR) are termed off-the-run FRAs.

Characteristics of Currency Forward Contracts

Under the terms of a **currency forward contract**, one party agrees to exchange a certain amount of one currency for a certain amount of another currency at a future date. This type of forward contract in practice will specify an exchange rate at which one party can buy a fixed amount of the currency underlying the contract. If we need to exchange 10 million Euros for U.S. dollars 60 days in the future, we might receive a quote of USD0.95. The forward contract specifies that we (the long) will purchase USD9.5 million for EUR10 million at settlement. Currency forward contracts can be deliverable or settled in cash. As with other forward contracts, the cash settlement amount is the amount necessary to compensate the party who would be disadvantaged by the actual change in market rates as of the settlement date. An example will illustrate this.

Example: Currency forwards

Gemco expects to receive EUR50 million three months from now and enters into a cash settlement currency forward to exchange these euros for U.S. dollars at USD1.03 per euro. If the market exchange rate is USD1.05 per euro at settlement, what is the amount of the payment to be received or paid by Gemco?

Answer:

Under the terms of the contract Gemco would receive:

EUR50 million × USD1.03 = USD51.5 million

Without the forward contract, Gemco would receive:

EUR50 million × USD1.05 = USD52.5 million

The counterparty would be disadvantaged by the difference between the contract rate and the market rate in an amount equal to the advantage that would have accrued to Gemco had they not entered into the currency forward.

Gemco must make a payment of USD1.0 million to the counterparty.

A direct calculation of the value of the long (USD) position at settlement is:

(USD1.03 – USD1.05) × EUR50 million = –USD1.0 million

KEY CONCEPTS

1. A forward contract specifies that the long will pay a certain amount at a specific future date to the short, who will deliver a certain amount of an asset (a financial asset, for our purposes). Default risk is the risk that the other party to the contract will not perform at settlement, since typically no money changes hands at the initiation of the contract.
2. A cash settlement forward contract does not require actual delivery of the underlying asset, but a cash payment at the settlement date from one counterparty to the other, based on the contract price and the market price of the asset at settlement.
3. Early termination of a forward contract can be accomplished by entering into a new forward contract with the opposite position, at the then-current expected future price. This early termination will fix the amount of the payment to be made or received at the settlement date. If this new forward is with a different counterparty than the original, there is credit or default risk to consider if either of the two counterparties fails in its obligation under the forward contract.
4. An end user of a forward contract is most often a corporation hedging an existing risk. Forward dealers, large banks, or brokerages originate forward contracts and take the long side in some contracts and the short side in others, profiting from the spread to compensate them for actual costs, bearing default risk, and any unhedged price risk.
5. An equity forward contract may be on a single stock, a customized portfolio, or a stock index, and removes uncertainty about equity prices at some future date. The contract can be written on a total return basis to include dividends, but most are based solely on an index value. Index forwards settle in cash based on the notional amount, the percentage difference between the index level at settlement, and the index level specified in the contract.
6. Forward contracts in which bonds are the underlying asset may be quoted in terms of the discount on zero-coupon bonds (T-bills) or in terms of the yield to maturity on coupon bonds. Forwards on corporate bonds must contain special provisions to deal with the possibility of default as well as any call or conversion features. Forward contracts may also be written on portfolios of fixed income securities or on bond indexes.
7. Eurodollar time deposits are USD-denominated short-term deposits (similar to certificates of deposit) purchased outside the U.S. from large money-center banks.
8. The London Interbank Offered Rate (LIBOR) is an international reference rate for Eurodollar deposits and is quoted for 30-day, 60-day, 90-day, 180-day, or 360-day (1-year) terms. Euribor is the equivalent for short-term Euro-denominated deposits—and for both, actual interest is based on the loan term as a percentage of a 360-day year.
9. Forward rate agreements (FRAs), although settled in cash, can be viewed as a forward contract for the long to get a loan from the short at a specific future date at a rate fixed in the contract. In fact, no loan is made and the contracts are settled in cash. They are described by the length of the contract and the term of the interest rate in the contract (e.g., 90-day LIBOR). If rates rise, the long receives a payment at settlement and the short receives a payment if the specified rate falls to a level below the contract rate.
10. The payment at settlement on an FRA is the present value of the difference in interest costs between a riskless loan at the market rate (usually LIBOR or Euribor) and one made at the rate specified in the contract. The difference in rates is multiplied by the notional amount of the contract to get the difference in interest due at the end of the loan term, then discounted back to the settlement date at the market rate of interest.
11. Currency forward contracts specify that one party will deliver a certain amount of one currency at the settlement date for a certain amount of another currency. Under a cash settlement option, a single cash payment is made based on the difference between the exchange rate fixed in the contract and the market-determined exchange rate at the settlement date.

Concept Checkers: Forward Markets and Contracts

1. The short in a deliverable forward contract:
   A. has no default risk.
   B. receives a payment at contract initiation.
   C. is obligated to deliver the specified asset.
   D. makes a cash payment to the long at settlement.

2. On the settlement date of a forward contract:
   A. the short may be required to sell the asset.
   B. the long must sell the asset or make a cash payment.
   C. at least one party must make a cash payment to the other.
   D. the long has the option to accept a payment or purchase the asset.

3. Which of the following statements regarding early termination of a forward contract is **TRUE**?
   A. There is no way to terminate a forward contract early.
   B. A party who enters into an offsetting contract to terminate has no risk.
   C. A party who terminates a forward contract early must make a cash payment.
   D. Early termination through an offsetting transaction with the original counterparty eliminates default risk.

4. A dealer in the forward contract market:
   A. cannot be a bank.
   B. may enter into a contract with another dealer.
   C. gets a small payment for each contract at initiation.
   D. typically specializes in either the long or short side of the contract.

5. Which of the following statements regarding equity forward contracts is **FALSE**?
   A. Equity forwards may be settled in cash.
   B. Dividends are never included in index forwards.
   C. Equity forwards are sometimes on custom stock portfolios.
   D. A short position in an equity forward could not hedge the risk of a purchase of that equity in the future.

6. Which of the following statements regarding forward contracts on T-bills is **TRUE**?
   A. The face value must be paid by the short at settlement.
   B. There is no default risk on these forwards because T-bills are government-backed.
   C. The long will receive a payment at settlement if the discount yield is above the forward yield.
   D. If short-term yields increase unexpectedly after contract initiation, the short will profit on the contract.

7. A Eurodollar time deposit:
   A. is priced on a discount basis.
   B. is only available in London.
   C. may be issued by a Japanese bank.
   D. is a certificate of deposit denominated in Euros.

8. One difference between LIBOR and Euribor is that:
   A. LIBOR is for London deposits.
   B. they are for different currencies.
   C. LIBOR is published daily, Euribor weekly.
   D. LIBOR is slightly higher due to default risk.

9. Which of the following statements regarding a LIBOR-based FRA is **TRUE**?
   A. The short will settle the contract by making a loan.
   B. FRAs can be based on interest rates for 30, 60, or 90-day periods.
   C. The contract rate will change with LIBOR over the term of the agreement.
   D. If LIBOR increases unexpectedly over the contract term, the long will be required to make a cash payment at settlement.

10. Consider a $2 million FRA with a contract rate of 5 percent on 60-day LIBOR. If 60-day LIBOR is 6 percent at settlement, the long will:
   A. pay $3,300.
   B. pay $3,333.
   C. receive $3,300.
   D. receive $3,333.

11. Party A has entered a currency forward contract to purchase €10 million at an exchange rate of $0.98 per €. At settlement, the exchange rate is $0.97 per euro. If the contract is settled in cash, party A will:
   A. make a payment of $100,000.
   B. receive a payment of $100,000.
   C. make a payment of €103,090.
   D. receive a payment of €103,090.

12. If the quoted discount yield on a 128-day, $1 million T-bill decreases from 3.15% to 3.07%, how much has the holder of the T-bill gained or lost?
   A. Lost $284.
   B. Gained $284.
   C. Lost $800.
   D. Gained $800.

13. 90-day LIBOR is quoted as 3.58%. How much interest would be owed at maturity for a 90-day loan of $1.5 million at LIBOR + 1.3%?
   A. $17,612.
   B. $32,925.
   C. $18,300.
   D. $73,200.

14. A company treasurer needs to borrow 10 million euros for 180 days, 60 days from now. The type of FRA and the position he should take to hedge the interest rate risk of this transaction are:

	FRA	Position
A.	2 × 6	Long
B.	2 × 6	Short
C.	2 × 8	Long
D.	2 × 8	Short

Answers – Concept Checkers: Forward Markets and Contracts

1. **C** The short in a forward contract is obligated to deliver the specified asset at the contract price on the settlement date. There is typically no payment made at contract initiation, and either party may have default risk if there is any probability that the counterparty may not perform under the terms of the contract.

2. **A** A forward contract may call for settlement in cash or for delivery of the asset but does not typically contain an option to do one or the other. Under a deliverable contract, the short is required to deliver the asset at settlement, not to make a cash payment.

3. **D** Terminating a forward contract early by entering into an offsetting forward contract with a different counterparty exposes a party to default risk. If the offsetting transaction is with the original counterparty, default risk is eliminated. No cash payment is required if an offsetting contract is used for early termination.

4. **B** Forward contracts dealers are commonly banks and large brokerage houses. They frequently enter into forward contracts with other dealers to offset long or short exposure. No payment is typically made at contract initiation.

5. **B** Index forward contracts may be written as total return contracts, which include dividends. Contracts may be written to settle in cash, be deliverable, or may be on custom portfolios. A *long* position is used to reduce the price risk of an expected future purchase.

6. **D** When short-term rates increase, T-bill prices fall and the short position will profit. The price of a T-bill prior to maturity is always less than its face value. There is default risk on the *forward*, even though the underlying asset is considered default-free.

7. **C** Eurodollar time deposits are U.S. dollar-denominated accounts with banks outside the U.S. and are quoted as an add-on yield rather than on a discount basis.

8. **B** LIBOR is for U.S. dollar-denominated accounts while Euribor is for euro-denominated accounts. Both can change daily and neither is location-specific. Differences in these rates are due to the different currencies involved, not differences in default risk.

9. **B** A LIBOR-based contract can be based on LIBOR for various terms. They are settled in cash and the contract rate is fixed for the life of the contract. The long will receive a payment when LIBOR is higher than the contract rate at settlement.

10. **C** $(0.06 - 0.05) \times (60 / 360) \times \$2 \text{ million} \times 1 / (1 + 0.06 / 6) = \$3{,}300.33$.

11. **A** ($0.98 – $0.97) × 10 million = $100,000 loss. The long, Party A, is obligated to buy euros at $0.98 when they are only worth $0.97 and must pay $0.01 × 10 million = $100,000.

12. **B** The actual discount has decreased by $(0.0315-0.0307)\times\frac{128}{360}=0.0284\%$ of $1,000,000 or $284. A decrease in the discount is an increase in value.

13. **C** $(0.0358 + 0.013)\left(\frac{90}{360}\right)1.5 \text{ million} = \$18{,}300$. Both LIBOR and any premium to LIBOR are quoted as annualized rates.

14. **C** This requires a long position in a 2 × 8 FRA.

The following is a review of the Derivative Investments principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

FUTURES MARKETS AND CONTRACTS

Study Session 16

EXAM FOCUS

Candidates should focus on the terminology of futures markets, how futures differ from forwards, the mechanics of margin deposits, and the process of marking to market. Limit price moves, delivery options, and the characteristics of the basic types of financial futures contracts are also likely exam topics. Learn the ways a futures position can be terminated prior to contract expiration and understand how cash settlement is accomplished by the final mark to market at contract expiration.

LOS 71.a: Identify the institutional features that distinguish futures contracts from forward contracts and describe the characteristics of futures contracts.

Futures contracts are very much like the forward contracts we learned about in the previous topic review. They are *similar* in that both:

- Can be either deliverable or cash settlement contracts.
- Are priced to have zero value at the time an investor enters into the contract.

Futures contracts *differ* from forward contracts in the following ways:

- Futures contracts trade on organized exchanges. Forwards are private contracts and do not trade.
- Futures contracts are highly standardized. Forwards are customized contracts satisfying the needs of the parties involved.
- A single clearinghouse is the counterparty to all futures contracts. Forwards are contracts with the originating counterparty.
- The government regulates futures markets. Forward contracts are usually not regulated.

Characteristics of Futures Contracts

Standardization. A major difference between forwards and futures is that futures contracts have standardized contract terms. Futures contracts specify the quality and quantity of goods that can be delivered, the delivery time, and the manner of delivery. The exchange also sets the minimum price fluctuation (which is called the tick size). For example, the basic price movement, or tick, for a 5,000-bushel grain contract is a quarter of a point (1 point = $0.01) per bushel, or $12.50 per contract. Contracts also have a daily price limit, which sets the maximum price movement allowed in a single day. For example, wheat cannot move more than $0.20 from its close the preceding day. The maximum price limits expand during periods of high volatility and are not in effect during the delivery month. The exchange also sets the trading times for each contract.

It would appear that these rules would restrict trading activity, but, in fact, they stimulate trading. Why? Standardization tells traders exactly what is being traded and the conditions of the transaction. *Uniformity promotes market liquidity.*

The purchaser of a futures contract is said to have gone long or taken a *long position*, while the seller of a futures contract is said to have gone short or taken a *short position*. For each contract traded, there is a buyer and a seller.

The long has contracted to buy the asset at the contract price at contract expiration, and the short has an obligation to sell at that price.

Clearinghouse. Each exchange has a *clearinghouse*. The clearinghouse guarantees that traders in the futures market will honor their obligations. The clearinghouse does this by splitting each trade once it is made and acting as the opposite side of each position. The clearinghouse acts as the buyer to every seller and the seller to every buyer. By doing this, the clearinghouse allows either side of the trade to reverse positions at a future date without having to contact the other side of the initial trade. This allows traders to enter the market knowing that they will be able to reverse their position. Traders are also freed from having to worry about the counterparty defaulting, since the counterparty is now the clearinghouse. In the history of U.S. futures trading, the clearinghouse has never defaulted on a trade.

Professor's Note: The terminology is that you "bought" bond futures if you entered into the contract with the long position. In my experience, this terminology has caused confusion for many candidates. You don't purchase the contract, you enter into it. You are contracting to buy an asset on the long side. "Buy" means take the long side and "sell" means take the short side in futures. (See Exam Flashback #1.)

LOS 71.b: Differentiate between margin in the securities markets and margin in the futures markets.

In securities markets, margin on a stock or bond purchase is a percentage of the market value of the asset. Initially, 50 percent of the stock purchase amount may be borrowed and the remaining amount, the equity in the account, must be paid in cash. There is interest charged on the borrowed amount, the margin loan. The margin percentage, the percent of the security value that is "owned," will vary over time and must be maintained at some minimum percentage of market value.

In the futures markets, margin is a performance guarantee. It is money deposited by both the long and the short. There is no *loan* involved and, consequently, no interest charges.

Each futures exchange has a clearinghouse. To safeguard the clearinghouse, the exchange requires traders to post margin and settle their accounts on a daily basis. Before trading, the trader must deposit funds (called margin) with their broker (who, in return, will post margin with the clearinghouse).

In securities markets, the cash deposited is paid to the seller of the security, with the balance of the purchase price provided by the broker. This is why the unpaid balance is a loan, with interest charged to the buyer who purchased on margin.

Initial and minimum margins in securities accounts are set by the Federal Reserve, although brokerage houses can require more. Initial and maintenance margins in the futures market are set by the clearinghouse and are based on historical daily price volatility of the underlying asset since margin is resettled daily in futures accounts. Margin in futures accounts is typically *much lower* as a percentage of the value of the assets covered by the futures contract. This means that the leverage, based on the actual cash required, is much higher for futures accounts.

LOS 71.c: Describe how a futures trade takes place.

In contrast to forward contracts in which a bank or brokerage is usually the counterparty to the contract, there is a buyer and a seller on each side of a futures trade. The futures exchange selects the contracts that will trade. The asset, the amount of the asset, and the settlement/delivery date are standardized in this manner (e.g., a June futures contract on 90-day T-bills with a face amount of $1 million). Each time there is a trade, the delivery price for that contract is the equilibrium price at that point in time, which depends on supply (by those wishing to be short) and demand (by those wishing to be long).

The mechanism by which supply and demand determine this equilibrium is open outcry at a particular location on the exchange floor called a "pit." Each trade is reported to the exchange so that the equilibrium price, at any point in time, is known to all traders.

LOS 71.d: Describe how a futures position may be closed out (i.e., offset) prior to expiration.

You may make a *reverse, or offsetting,* trade in the futures market. This is similar to the way we described exiting a forward contract prior to expiration. With futures, however, the other side of your position is held by the clearinghouse—if you make an exact opposite trade (maturity, quantity, and good) to your current position, the clearinghouse will net your positions out, leaving you with a zero balance. This is how most futures positions are settled.

The contract price can differ between the two contracts. If you initially are long one contract at $370 per ounce of gold and subsequently sell (take the short position in) an identical gold contract when the price is $350/oz., $20 times the number of ounces of gold specified in the contract will be deducted from the margin deposit(s) in your account. The sale of the futures contract ends the exposure to future price fluctuations on the first contract. Your position has been *reversed,* or *closed out,* by a *closing* trade.

LOS 71.e: Define initial margin, maintenance margin, variation margin, and settlement price.

Initial margin is the money that must be deposited in a futures account before any trading takes place. It is set for each type of underlying asset. Initial margin per contract is relatively low and equals about one day's maximum price fluctuation on the total value of the contract's underlying asset.

Maintenance margin is the amount of margin that must be maintained in a futures account. If the margin balance in the account falls below the maintenance margin due to a change in the contract price for the underlying asset, additional funds must be deposited to bring the margin balance back up to the initial margin requirement.

This is in contrast to equity account margins, which require investors only to bring the margin percentage up to the maintenance margin, not back to the initial margin level.

Variation margin is the funds that must be deposited into the account to bring it back to the initial margin amount. If account margin exceeds the initial margin requirement, funds can be withdrawn or used as initial margin for additional positions.

The **settlement price** is analogous to the closing price for a stock but is not simply the price of the last trade. It is an average of the prices of the trades during the last period of trading, called the closing period, which is set by the exchange. This feature of the settlement price prevents manipulation by traders. The settlement price is used to make margin calculations at the end of each trading day. *(See Exam Flashback #2.)*

LOS 71.f: Describe the process of marking to market and compute the margin balance, given the previous day's balance and the new futures price.

Marking to market is the process of adjusting the margin balance in a futures account each day for the change in the value of the contract assets from the previous trading day, based on the new settlement price.

The futures exchanges can require a mark to market more frequently (than daily) under extraordinary circumstances.

Computing the Margin Balance

Example: Margin balance

Consider a long position of five July wheat contracts, each of which covers 5000 bushels. Assume that the contract price is $2.00 and that each contract requires an initial margin deposit of $150 and a maintenance margin of $100. The total initial margin required for the five-contract trade is $750. The maintenance margin for the account is $500. **Compute** the margin balance for this position after a 2-cent decrease in price on day 1, a 1-cent increase in price on day 2, and a 1-cent decrease in price on day 3.

Answer:

Each $0.01 change in the price of wheat for this contract represents a gain or loss of $50, or $250 for the five contracts. That is: (0.01)(5)(5000) = $250.00. Each contract is for 5,000 bu. so that a price change of $0.01 per bu. changes the contract value by $50.

The table in Figure 1 illustrates the change in the margin balance as the price of this contract changes each day. Note that the initial balance is the initial margin requirement of $750 and that the required deposit is based on the previous day's price change.

Figure 1: Margin Balances

Day	*Required Deposit*	*Price/bushel*	*Daily Change*	*Gain/Loss*	*Balance*
0 (Purchase)	$750	$2.00	0	0	$750
1	0	$1.98	–$0.02	–$500	$250
2	$500	$1.99	+$0.01	+$250	$1,000
3	0	$1.98	–$0.01	–$250	$750

At the close on day 1, the margin balance has gone below the minimum or maintenance margin level of $500. Therefore, a deposit of $500 is required to bring the margin back to the initial margin level of $750.

LOS 71.g: Explain price limits, limit move, limit up, limit down, and locked limit.

Many futures contracts have **price limits**, which are exchange-imposed limits on how much the contract price can change from the previous day's settlement price. Exchange members are prohibited from executing trades at prices outside these limits. If the (equilibrium) price at which traders would willingly trade, is above the upper limit or below the lower limit, trades cannot not take place.

Consider a futures contract that has daily price limits of 2 cents and settled the previous day at $1.04. If, on the following trading day, traders wish to trade at $1.07 because of changes in market conditions or expectations, no trades will take place. The settlement price will be reported as $1.06 (for the purposes of marking to market). The contract will be said to have made a **limit move**, and the price is said to be **limit up** (from the previous day). If market conditions had changed such that the price at which traders are willing to trade is below $1.02, $1.02 will be the settlement price and the price is said to be **limit down**. If trades cannot take place because of a limit move, either up or down, the price is said to be **locked limit**, since no trades can take place and traders are "locked" into their existing positions.

LOS 71.h: Describe how a futures contract can be terminated by a close-out (i.e., offset) at expiration, delivery, an equivalent cash settlement, or an exchange-for-physicals.

There are four **ways to terminate a futures contract**:

- A short can terminate the contract by delivering the goods, a long by accepting delivery and paying the contract price to the short. This is called *delivery.* The location for delivery (for physical assets), terms of delivery, and details of exactly what is to be delivered are all specified in the contract. Deliveries represent less than 1 percent of all contract terminations.
- In a **cash-settlement contract**, delivery is not an option. The futures account is marked-to-market based on the settlement price on the last day of trading.
- A position can be terminated by making a close-out or *offsetting trade* in the futures market as previously described. Since the other side of your position is held by the clearinghouse, if you make an exact opposite trade (maturity, quantity, and asset) to your current position, the clearinghouse will net your positions out, leaving you with no future obligation. This is how most futures positions are terminated.
- A position may also be settled through an *exchange for physicals (EFP).* Here, you find a trader with an opposite position to your own and deliver the goods and settle up between yourselves, off the floor of the exchange (called an ex-pit transaction). This is the sole exception to the federal law that requires that all trades take place on the floor of the exchange. You must then contact the clearinghouse and tell them what happened. An exchange for physicals differs from a delivery in that the traders actually exchange the goods, the contract is not closed on the floor of the exchange, and the two traders privately negotiate the terms of the transaction. Regular delivery involves only one trader and the clearinghouse.

LOS 71.i: Explain delivery options in futures contracts.

Some futures contracts grant **delivery options** to the short; options on what, where, and when to deliver. Some Treasury bond contracts give the short a choice of several bonds that are acceptable to deliver and options as to when to deliver during the expiration month. Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract.

LOS 71.j: Distinguish among scalpers, day traders, and position traders.

The terms scalpers, day traders, and position traders refer to participants in the futures markets on the floor of the exchange (in the pits).

- **Scalpers** try to make money by buying at the bid and selling at the ask. They hold positions for very short periods of time, possibly less than a minute. They are not speculating on the direction of the price movement. Scalpers profit opportunistically from trades and provide liquidity to the market by their trading in the process.
- **Day traders** attempt to profit by anticipating the direction of short-term price movements and, as their name implies, do not carry open positions past the end of the trading day.
- **Position traders** are traders who attempt to profit from anticipating the direction of price moves and hold positions longer, overnight or for days.

All of these types of traders are members who hold seats on the exchange, do not execute trades for public investors (nonmembers of the exchange), and provide liquidity to the futures markets.

LOS 71.k: Describe the characteristics of the following types of futures contracts: Treasury bill, Eurodollar, Treasury bond, stock index, and currency.

Treasury bill futures contracts are based on a \$1 million face value 90-day (13-week) T-bill and settle in cash. The price quotes are 100 minus the annualized discount in percent on the T-bills.

A price quote of 98.52 represents an annualized discount of 1.48 percent, an actual discount from face of $0.0148 \times (90 / 360) = 0.0037$, and a "delivery" price of $(1 - 0.0037) \times 1$ million = \$996,300.

Professor's Note: One of the first things a new T-bill futures trader learns is that each change in price of 0.01 in the price of a T-bill futures contract is worth \$25. If you took a long position at 98.52 and the price fell to 98.50, your loss is \$50 per contract. Since Eurodollar contracts on 90-day LIBOR are the same size and priced in a similar fashion, a price change of 0.01 represents a \$25 change in value for these as well.

T-bill futures contracts are not as important as they once were. Their prices are heavily influenced by U.S. Federal Reserve operations and overall monetary policy. T-bill futures have lost importance in favor of Eurodollar futures contracts which represent a more free-market and more global measure of short-term interest rates to top quality borrowers for U.S. \$-denominated loans.

Eurodollar futures are based on 90-day LIBOR, which is an add-on yield. By convention, however, the price quotes follow the same convention as T-bills and are calculated as (100 – annualized LIBOR in percent). These contracts settle in cash and the minimum price change is one "tick," which is a price change of 0.0001 = 0.01%, representing \$25 per \$1 million contract.

Treasury bond futures contracts:

- Are traded for Treasury bonds with maturities greater than 15 years.
- Are a deliverable contract.
- Have a face value of \$100,000.
- Are quoted as a percent and fractions of one percent (measured in 1/32nds) of face value.

The short in a Treasury bond futures contract has the option to deliver any of several bonds which will satisfy the delivery terms of the contract. This is called a delivery option and is valuable to the short.

Each bond is given a *conversion factor* which is used to adjust the long's payment at delivery so that the more valuable bonds receive a higher payment. These factors are multipliers for the futures price at settlement. The long pays the futures price at expiration times the conversion factor.

Stock index futures. The most popular stock index future is the S&P 500 Index Future that trades in Chicago. Settlement is in cash and is based on a multiplier of 250.

The value of a contract is 250 times the level of the index stated in the contract. With an index level of 1000, the value of each contract is \$250,000. Each index point in the futures price represents a gain or loss of \$250 per contract. A long stock index futures position on S&P 500 index futures at 1,051 would show a gain of \$1,750 in the trader's account if the index were 1,058 at the settlement date (\$250 × 7 = \$1,750). A smaller contract is traded on the same index and has a multiplier of 50.

Futures contracts covering several other popular indexes are traded, and the pricing and contract valuation are the same, although the multiplier can vary from contract to contract. *(See Exam Flashback #3.)*

Currency futures. The currency futures market is smaller in volume than the forward currency market we described in the previous topic review. In the U.S., currency contracts trade on the euro (EUR), Mexican peso (MXP), and yen (JPY), among others. Contracts are set in units of the foreign currency and the price is stated in USD/unit. The size of the peso contract is MXP500,000 and the euro contract is on EUR125,000. A change in the price of the currency unit of USD0.0001 translates into a gain or loss of USD50 on a MXP500,000 unit contract and USD12.50 on a EUR125,000 unit contract.

Key Concepts

1. Compared to forward contracts, futures contracts:
    - Are more liquid and trade on exchanges.
    - Do not have counterparty risk; the clearinghouse acts as counterparty.
    - Have lower transactions costs.
    - Require margin deposits and are marked to market daily.
    - Are standardized contracts and can not be customized.
2. Futures contracts have a buyer (the long) and a seller (the short), have prices determined by open outcry on the exchange floor, and are standardized as to asset quantity, quality, settlement dates, and delivery requirements.
3. Futures margin deposits are not loans, but deposits to insure performance under the terms of the contract.
4. A long or short position in a futures contract can be closed out by an offsetting trade (i.e., entering into an opposite position in the same contract).
5. Initial margin is the deposit required to initiate a futures position; maintenance margin is the minimum margin and balances below this will trigger a requirement to post additional (variation) margin.
6. Marking to market is the process of adding gains to or subtracting losses from the margin account daily, based on the change in settlement (closing) prices from one day to the next.
7. The mark to market adjustment is calculated as the change in contract price per asset unit, times the quantity specified in the contract, times the number of contracts, and is added to (subtracted from) the margin account of the long (short).
8. Trades cannot take place at prices that differ from the previous day's settlement prices by more than the price limit and are said to be limit down (up) when the new equilibrium price is below (above) the minimum (maximum) price for the day.
9. A futures position can be terminated by:
    - An offsetting trade.
    - Cash payment at expiration.
    - Delivery of the asset specified in the contract.
    - An exchange for physicals (asset delivery off the exchange).
10. The short in a futures contract may have valuable delivery options that allow a choice of the exact asset to be delivered, the location of delivery, or the date of delivery.
11. Three types of futures market participants are:
    - Scalpers, who profit from buying at the bid and selling at the ask.
    - Day traders, who seek short term profits from intraday price moves.
    - Position traders who attempt to profit from longer-term price moves.
12. Treasury bill contracts are quoted at 100 – annualized discount %, are for T-bills with a face value of $1,000,000, and settle in cash.
13. Eurodollar futures contracts are for a face value of $1,000,000, are quoted as 100 – annualized 90-day LIBOR, and settle in cash.
14. Treasury bond contracts are for a face value of $100,000, give the short the choice of bonds to deliver, and use conversion factors to adjust the contract price for the bond that is delivered.
15. Stock index futures have a multiplier that is multiplied by the index to calculate the contract value and settle in cash.
16. Currency futures are for delivery of standardized amounts of foreign currency.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #89 from '96 actual exam, and '98 sample exam.

Futures contracts *differ* from forward contracts in the following ways:

I. Futures contracts are standardized.
II. For futures, performance of each party is guaranteed by a clearinghouse.
III. Futures contracts require a daily settling of any gains or losses.

A. I and II only.
B. I and III only.
C. II and III only.
D I, II, and III.

Exam Flashback # 2

Source: Question #86 from '99–'03 sample exams.

A silver futures contract requires the seller to deliver 5,000 Troy ounces of silver. An investor sells one July silver futures contract at a price of $8 per ounce, posting a $2,025 initial margin. If the required maintenance margin is $1,500, the price per ounce at which the investor would first receive a maintenance margin call is *closest* to:
A. $5.92.
B. $7.89.
C. $8.11.
D. $10.80.

Exam Flashback # 3

Source: Question #76 from '93 and '96 actual exam.

On the maturity date, stock index futures contracts require delivery of:
A. common stock.
B. common stock plus accrued dividends.
C. Treasury bills.
D. cash.

Concept Checkers: Futures Markets and Contracts

1. Which of the following statements is **FALSE**?
   A. Hedgers trade to reduce some preexisting risk exposure.
   B. The clearinghouse guarantees that traders in the futures market will honor their obligations.
   C. If an account rises to or exceeds the maintenance margin, then the trader must deposit variation margin.
   D. In marking to market, any losses for the day are removed from the trader's account and any gains are added to the trader's account.

2. The daily process of adjusting the margin in a futures account is called:
   A. initial margin.
   B. variation margin.
   C. marking to market.
   D. maintenance margin.

3. A trader buys (takes a long position in) a T-bill futures contract ($1 million face value) at 98.14 and closes it out at a price of 98.27. On this contract the trader has:
   A. lost $325.
   B. gained $325.
   C. lost $1,300.
   D. gained $1,300.

4. In the futures market, a contract does not trade for two days because trades are not permitted at the equilibrium price. The market for this contract is:
   A. limit up.
   B. suspended.
   C. limit down.
   D. locked limit.

5. The existence of a delivery option with respect to Treasury bond futures means that the:
   A. short can choose which bond to deliver.
   B. short has the option to settle in cash or by delivery.
   C. long chooses which of a number of bonds will be delivered.
   D. long can choose to deliver the bond or require cash settlement.

6. Assume the holder of a long futures position negotiates privately with the holder of a short futures position to accept delivery to close out both the long and short positions. Which of the following statements about the transaction is *most correct*? The transaction is:
   A. also known as delivery.
   B. also known as an exchange of physicals.
   C. the most common way to close a futures position.
   D. not permitted; exchange rules require that all transactions take place on the floor of the exchange.

7. A conversion factor in a Treasury bond contract is:
   A. used to adjust the number of bonds to be delivered.
   B. multiplied by the face value to determine the delivery price.
   C. multiplied by the futures price to determine the delivery price.
   D. divided into the futures price at contract initiation for margin calculations.

8. Three 125,000 euro futures contracts are sold at a price of $1.0234. The next day the price settles at $1.0180. The mark to market for this account changes the previous day's margin by:
   A. + $675.
   B. – $675.
   C. + $2,025.
   D. – $2,025.

9. Which of the following is a futures exchange member who can execute public orders?
   A. Scalper.
   B. Day trader.
   C. Floor broker.
   D. Position trader.

10. In the futures market, the clearinghouse does all of the following **EXCEPT**:
    A. decide which contracts will trade.
    B. set initial and maintenance margins.
    C. act as the counterparty to every trade.
    D. receive margin deposits from brokers.

11. Funds deposited to meet a margin call are termed:
    A. daily margin.
    B. loan payments
    C. settlement costs.
    D. variation margin.

12. Compared to forward contracts, futures contracts are all of the following **EXCEPT**:
    A. more liquid.
    B. standardized.
    C. larger in size.
    D. less subject to default risk.

Answers – Exam Flashbacks

1. **D** Compared to forwards, futures: (1) are more liquid, (2) do not have counterparty risk, (3) have lower transactions costs, (4) require margin and must be marked to market, (5) cannot be customized.

   Professor's Note: Comparing two securities that can be used to produce the same result and explaining their differences has been a popular CFA exam question at all levels.

2. **C** A good way to deal with futures margins and mark to market calculations is to first calculate the movement in the contract value for a one-unit change in price—a dollar, a percent, a basis point, whatever fits the contract. One cent seems to fit here. A one-cent change in the price of silver means a \$50 change on 5,000 ounces. To lose more than \$2,025 – \$1,500 = \$525, \$0.11 will do it since 11 × 50 is 550.

   Professor's Note: ALWAYS go back and check to see if you have the long or short position right after you mark your answer to make sure you had the price move in the right direction.

3. **D** Index futures settle in cash. The mechanics are simple, your account is marked to market at the settlement price on the last day of trading. Your "cash settlement" is already in your account when it's marked to market; whether you have gains or losses, they have been accumulating daily through margin adjustment over the life of the contract.

Answers – Concept Checkers: Futures Markets and Contracts

1. **C** If an account rises to or exceeds the maintenance margin, no payment needs to be made, and the trader has the option to remove the excess funds from the account. Only if an account falls below the maintenance margin does variation margin need to be paid to bring the level of the account back up to the level of the initial margin.

2. **C** The *process* is called marking to market. Variation margin is the funds that must be deposited when marking to market draws the margin balance below the maintenance margin.

3. **B** The price is quoted as (one minus the annualized discount) in percent. Remember that the gains and losses on T-bill and Eurodollar futures are \$25 per basis point of the price quote. The price is up 13 ticks and 13 × \$25 is a gain of \$325 for a long position.

4. **D** This describes the situation when the equilibrium price is either above or below the prior day's settle price by more than the permitted (limit) daily price move. We do not know whether it is limit up or limit down. Trading is not suspended; trades can take place if they are within the band of the limits.

5. **A** The short has the option to deliver any of a number of permitted bonds. The delivery price is adjusted by a conversion factor that is calculated for each permitted bond.

6. **B** When the holder of a long position negotiates directly with the holder of the short position to accept delivery of the underlying commodity to close out both positions, this is called an *exchange for physicals*. (This is a private transaction that occurs *ex-pit* and is one exception to the federal law that all trades take place on the exchange floor.) Note that the exchange for physicals differs from an offsetting trade in which no delivery takes places, and also differs from delivery in which the commodity is simply delivered as a result of the futures expiration with no secondary agreement. Most futures positions are settled by an *offsetting trade*.

7. **C** It adjusts the delivery price based on the futures price at contract expiration.

8. **C** (1.0234 – 1.0180) × 125,000 × 3 = \$2,025. The contracts were sold and the price declined, so the adjustment is an addition to the account margin.

9. **C** Traders trade with other exchange members on the exchange floor. Brokers execute orders for those off the exchange.

10. **A** The exchange determines which contracts will trade.

11. **D** When insufficient funds exist to satisfy margin requirements, a variation margin must be posted.

12. **C** Size is not one of the things that distinguishes forwards and futures, although the contract size of futures is standardized, whereas forwards are customized for each party.

The following is a review of the Derivative Investments principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

Option Markets and Contracts

Study Session 16

Exam Focus

This derivatives review introduces options, describes their terms and trading, and provides derivations of several options valuation results. Candidates should spend some time understanding how the payoffs on several types of options are determined. This includes options on stocks, bonds, stock indexes, interest rates, currencies, and futures. The assigned material on establishing upper and lower bounds is extensive, so it should not be ignored. Candidates must learn at least one of the put-call parity relations and how to construct an arbitrage strategy. The notation, formulas, and relations may seem daunting, but if you put in the time to understand what the notation is saying (and why), you can master the important points.

LOS 72.a: Identify the basic elements and describe the characteristics of option contracts.

An **option contract** gives its owner the right, but not the legal obligation, to conduct a transaction involving an underlying asset at a predetermined future date (the exercise date) and at a predetermined price (the exercise or strike price). Options give the option buyer the right to decide whether or not the trade will eventually take place. The seller of the option has the obligation to perform if the buyer exercises the option.

- The owner of a *call option* has the right to purchase the underlying asset at a specific price for a specified time period.
- The owner of a *put option* has the right to sell the underlying asset at a specific price for a specified time period.

For every owner of an option, there must be a seller. The seller of the option is also called the *option writer.* There are four possible options positions:

- Long call: the buyer of a call option—has the right to buy an underlying asset.
- Short call: the writer (seller) of a call option—has the obligation to sell the underlying asset.
- Long put: the buyer of a put option—has the right to sell the underlying asset.
- Short put: the writer (seller) of a put option—has the obligation to buy the underlying asset.

To acquire these rights, owners of options must buy them by paying a price called the *option premium* to the seller of the option.

Listed stock option contracts trade on exchanges and are normally for 100 shares of stock. After issuance, stock option contracts are adjusted for stock splits but not cash dividends.

To see how an option contract works, consider the stock of ABC Company. It sells for $55 and has a call option available on it that sells for a premium of $10. This call option has an exercise price of $50 and has an expiration date in five months. The *exercise price* of $50 is often called the option's *strike price.*

Professor's Note: The option premium is simply the price of the option. Please do not confuse this with the exercise price of the option, which is the price at which the underlying asset will be bought/sold if the option is exercised.

If the ABC call option is purchased for \$10, the buyer can purchase ABC stock from the option seller over the next five months for \$50. The seller, or writer, of the option gets to keep the \$10 premium no matter what the stock does during this time period. If the option buyer exercises the option, the seller will receive the \$50 strike price and must deliver to the buyer a share of ABC stock. If the price of ABC stock falls to \$50 or below, the buyer is not obligated to exercise the option. Note that option holders will only exercise their right to act if it is profitable to do so. The option writer, however, has an obligation to act at the request of the option holder.

A put option on ABC stock is the same as a call option except the buyer of the put (long position) has the right to sell a share of ABC for \$50 at any time during the next five months. The put writer (short position) has the obligation to buy ABC stock at the exercise price in the event that the option is exercised.

The owner of the option is the one who decides whether to exercise the option or not. If the option has value, the buyer may either exercise the option or sell the option to another buyer in the secondary options market.

LOS 72.b: Define European option, American option, moneyness, payoff, intrinsic value, and time value and differentiate between exchange-traded options and over-the-counter options.

- **American options** may be exercised at any time up to and including the contract's expiration date.
- **European options** can be exercised only on the contract's expiration date.

Professor's Note: The name of the option does not imply where the option trades—they are just names.

At expiration, an American option and a European option on the same asset with the same strike price are identical. They may either be exercised or allowed to expire. Before expiration, however, they are different and may have different values, so you must distinguish between the two.

If two options are identical (maturity, underlying stock, strike price, etc.) in all ways, except that one is a European option and the other is an American option, the value of the American option will equal or exceed the value of the European option. Why? The early exercise feature of the American option gives it more flexibility, so it should be worth at least as much and possibly more. *(See Exam Flashback #1.)*

Moneyness refers to whether an option is *in-the-money* or *out-of-the-money*. If immediate exercise of the option would generate a positive payoff, it is in-the-money. If immediate exercise would result in a loss (negative payoff), it is out-of-the-money. When the current asset price equals the exercise price, exercise will generate neither a gain nor loss, and the option is *at-the-money*.

The following describe the conditions for a **call option** to be in-, out-of-, or at-the-money.

- *In-the-money call options.* If $S - X > 0$, a call option is in-the-money. $S - X$ is the amount of the payoff a call holder would receive from immediate exercise, buying a share for X and selling it in the market for a greater price S.
- *Out-of-the-money call options.* If $S - X < 0$, a call option is out-of-the-money.
- *At-the-money call options.* If $S = X$, a call option is said to be at-the-money.

The following describe the conditions for a **put option** to be in-, out-of-, or at-the-money.

- *In-the-money put options.* If $X - S > 0$, a put option is in-the-money. $X - S$ is the amount of the payoff from immediate exercise, buying a share for S and selling it in the market for a greater price X.
- *Out-of-the-money put options.* When the stock's price is greater than the strike price, a put option is said to be out-of-the-money. If $X - S < 0$, a put option is out-of-the-money.
- *At-the-money put options.* If $S = X$, a put option is said to be at-the-money.

Example: Moneyness

Consider a July 40 call and a July 40 put, both on a stock that is currently selling for \$37/share. **Calculate** how much these options are, in- or out-of-the money.

Professor's Note: A July 40 call is a call option with an exercise price of \$40 and an expiration date in July.

Answer:

The call is \$3 out-of-the-money because $S - X = -\$3.00$.
The put is \$3 in-the-money because $X - S = \$3.00$.

An option's **intrinsic value** is the amount by which the option is in-the-money. It is the amount that the option owner would receive if the option were exercised. An option has zero intrinsic value if it is at-the-money or out-of-the-money, regardless of whether it is a call or a put option.

Let's look at the value of a call option *at expiration*. If the expiration date price of the stock exceeds the strike price of the option, the call owner will exercise the option and receive S – X. If the price of the stock is less than or equal to the strike price, the call holder will let the option expire and get nothing.

The *intrinsic value of a call* option is the greater of (S – X) or 0. That is:

$$C = \text{Max}[0, S - X]$$

Similarly, the *intrinsic value of a put* option is (X – S) or 0, whichever is greater. That is:

$$P = \text{Max}[0, X - S]$$

Example: Intrinsic value

Consider a call option with a strike price of \$50. **Compute** the intrinsic value of this option for stock prices of \$55, \$50, and \$45.

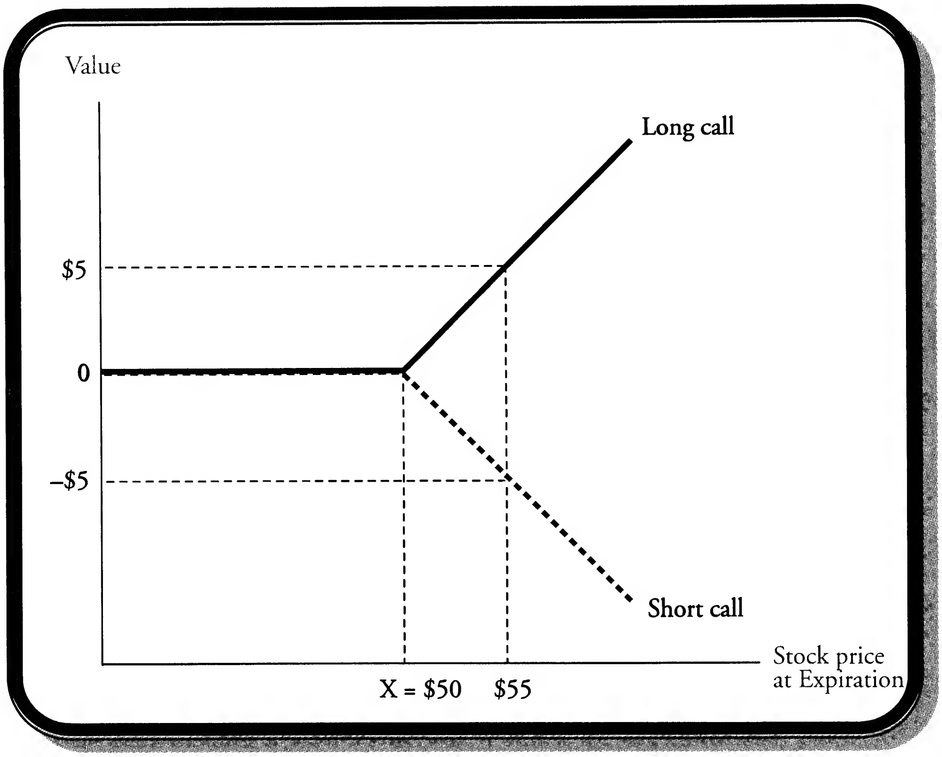
Answer:

Stock price = \$55: $C = \text{Max}[0, S - X] = \text{Max}[0, (55 - 50)] = \$5$
Stock price = \$50: $C = \text{Max}[0, S - X] = \text{Max}[0, (50 - 50)] = \$0$
Stock price = \$45: $C = \text{Max}[0, S - X] = \text{Max}[0, (45 - 50)] = \$0$

Notice that at expiration, if the stock is worth \$50 or below, the call option is worth \$0. Why? Because a rational option holder will not exercise the call option and take the loss. This "one-sided" feature of call options is illustrated in the option payoff diagram presented in Figure 1 for the call option we have used in this example.

Professor's Note: Option payoff diagrams are commonly used tools to illustrate the value of an option at expiration.

Figure 1: Call Option Payoff Diagram

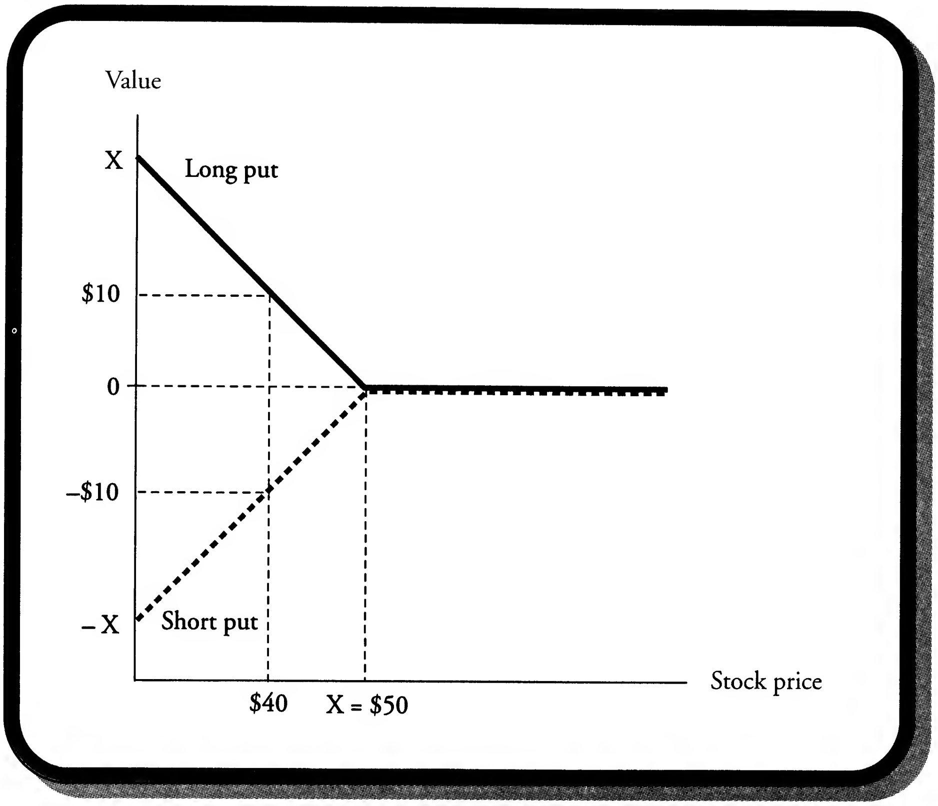


As indicated in Figure 1, the expiration date payoff to the owner is either zero or the amount that the option is in-the-money. For a call option writer (seller), the payoff is either zero or minus the amount it is in-the-money. There are no positive payoffs for an option writer. The option writer receives the premium and takes on the obligation to pay whatever the call owner gains. With reference to Figure 1, you should make the following observations:

- The payoff to a long call position (the solid line) is a flat line which angles upward to the right at a 45 degree angle from the strike price, *X*.
- The payoff to the writer of a call (dotted line), is a flat line which angles downward to the right at a 45 degree angle from the strike price, *X*.
- Options are a zero-sum game. If you add the long call option's payoff to the short option's payoff, you will get a net payoff of zero.
- At a stock price of $55, the payoff to the long is $5, which is a $5 loss to the short.

Similar to our payoff diagram for a call option, Figure 2 illustrates the at-expiration payoff values for a put option. As indicated here, if the price of the stock is less than the strike price, the put owner will exercise the option and receive (X – S). If the price of the stock is greater than or equal to the strike price, the put holder will let the put option expire and get nothing (0). At a stock price of $40, the payoff on a long put is $10; the seller of the put (the short) would have a negative payoff since he must buy the stock at $50 and receive stock worth $40.

Figure 2: Put Option Payoff Diagram



The **time value** of an option is the amount by which the option premium exceeds the intrinsic value and is sometimes called the speculative value of the option. This relationship can be written as:

option value = intrinsic value + time value

As we discussed earlier, the intrinsic value of an option is the amount by which the option is in-the-money. At any point during the life of an options contract, its value will typically be greater than its intrinsic value. This is because there is some probability that the stock price will change in an amount that gives the option a positive payoff at expiration greater than the (current) intrinsic value. Recall that an option's intrinsic value (to a buyer) is the amount of the payoff at expiration and is bounded by zero. When an option reaches expiration there is no "time" remaining and the time value is zero. For American options and in most cases for European options, the longer the time to expiration, the greater the time value and, other things equal, the greater the option's premium (price).

Exchange-Traded Options vs. Over-the-Counter Options

Exchange-traded or listed options are regulated, standardized, liquid, and backed by the Options Clearing Corporation for Chicago Board Options Exchange transactions.

Over-the-counter (OTC) options on stocks for the retail trade all but disappeared with the growth of the organized exchanges in the 1970s. There is now, however, an active market in OTC options on currencies, swaps, and equities, primarily for institutional buyers. Like the forward market, the OTC options market is largely unregulated, consists of custom options, involves counterparty risk, and is facilitated by dealers in much the same way forwards markets are.

LOS 72.c: Identify the different types of options in terms of the underlying instruments.

The three types of options we consider are: (1) financial options, (2) options on futures, and (3) commodity options.

Financial options include equity options and other options based on stock indexes, Treasury bonds, interest rates, and currencies. The strike price for financial options can be in terms of yield-to-maturity on bonds, an index level, or an exchange rate for *foreign currency options*. LIBOR-based *interest rate options* have payoffs based on the difference between LIBOR at expiration and the strike rate in the option.

Bond options are most often based on Treasury bonds because of their active trading. There are relatively few listed options on bonds—most are over-the-counter options. Bond options can be deliverable or settle in cash. The mechanics of bond options are like those of equity options, but are based on bond prices and a specific face value of the bond. The buyer of a call option on a bond will gain if interest rates fall and bond prices rise. A put buyer will gain when rates rise and bond prices fall.

Index options settle in cash, nothing is delivered, and the payoff is made directly to the option holder's account. The payoff on an index call (long) is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the strike price), multiplied by the *contract multiplier*. An equal amount will be deducted from the account of the index call option writer.

Example: Index options

Assume that you own a call option on the S&P 500 Index with an exercise price equal to 950. The multiplier for this contract is 250. **Compute** the payoff on this option assuming that the index is 962 at expiration.

Answer:

This is a call, so the expiration date payoff is (962 – 950) × \$250 = \$3,000.

Options on futures, sometimes called futures options, give the holder the right to buy or sell a specified futures contract on or before a given date at a given futures price, the strike price.

- *Call options* on futures contracts give the holder the right to enter into the long side of a futures contract at a given futures price. Assume that you hold a call option on a bond future at 98 (percent of face) and at expiration the futures price on the bond contract is 99. By exercising the call, you take on a long position in the futures contract, and the account is immediately marked to market based on the settlement price. Your account would be credited with cash in an amount equal to 1 percent (99 – 98) of the face value of the bonds covered by the contract. The seller of the exercised call will take on the short position in the futures contract and the mark to market value of this position will generate the cash deposited to your account.
- *Put options* on futures contracts give the holder the option to take on a short futures position at a futures price equal to the strike price. The writer has the obligation to take on the opposite (long) position if the option is exercised.

Commodity options give the holder the right to either buy or sell a fixed quantity of some physical asset at a fixed (strike) price.

LOS 72.d: Compare and contrast interest rate options to forward rate agreements (FRAs).

Interest rate options are similar to the stock options except that the exercise price is an interest rate and the underlying asset is a reference rate such as LIBOR. Interest rate options are also similar to FRAs because there is no deliverable asset. Instead they are settled in cash, in an amount that is based on a notional amount and the spread between the strike rate and the reference rate. Most interest rate options are European options.

To see how interest rate options work, consider a long position in a LIBOR-based interest rate call option with a notional amount of $1,000,000 and a strike rate of 5 percent. For our example, let's assume that this option is costless. If at expiration, LIBOR is greater than 5 percent, the option can be exercised and the owner will receive $1,000,000 × (LIBOR – 5%). If LIBOR is less than 5 percent, the option expires worthless and the owner receives nothing.

Now, let's consider a short position in a LIBOR-based interest rate put option with the same features as the call that we just discussed. Again, the option is assumed to be costless, with a strike rate of 5 percent and notional amount of $1,000,000. If at expiration, LIBOR falls below 5 percent, the option writer (short) must pay the put holder an amount equal to $1,000,000 × (5% – LIBOR). If at expiration, LIBOR is greater than 5 percent, the option expires worthless and the put writer makes no payments.

Notice the one-sided payoff on these interest rate options. The long call receives a payoff when LIBOR exceeds the strike rate and receives nothing if LIBOR is below the strike rate. On the other hand, the short put position makes payments if LIBOR is below the strike rate, and makes no payments when LIBOR exceeds the strike rate.

The combination of the long interest rate call option plus a short interest rate put option has the same payoff as a forward rate agreement (FRA). To see this, consider the fixed-rate payer in a 5 percent fixed-rate, $1,000,000 notional, LIBOR-based FRA. Like our long call position, the fixed-rate payer will receive $1,000,000 × (LIBOR – 5%). And, like our short put position, the fixed rate payer will pay $1,000,000 × (5% – LIBOR).

Professor's Note: For the exam, you need to know that a long interest rate call combined with a short interest rate put has the same payoff as a long position in an FRA.

LOS 72.e: Explain how option payoffs are determined, and show how interest rate option payoffs differ from the payoffs of other types of options.

Calculating the payoff for a stock option, or other type of option with a monetary-based exercise price, is straightforward. At expiration, a call owner receives any amount by which the asset price exceeds the strike price, and zero otherwise. The holder of a put will receive any amount that the asset price is below the strike price at expiration, and zero otherwise.

While bonds are quoted in terms of yield-to-maturity, T-bills in discount yield, indexes in index points, and currencies as an exchange rate, the same principle applies. That is, in each case, to get the payoff per unit of the relevant asset, we need to translate the asset value to a dollar value and the strike price (or rate, or yield) to a dollar strike price. We can then multiply this payoff times however many units of the asset are covered by the options contract.

- For a stock index option, we saw that these dollar values were obtained from multiplying the index level and the strike level by the multiplier specified in the contract. The resulting dollar payoffs are per contract.
- The payoff on options on futures is the cash the option holder receives when he exercises the option and the resulting futures position is marked to market.

The **payoffs on interest rate options** are different. For example, a call option based on 90-day LIBOR makes a payment based on a stated notional amount and the difference between 90-day LIBOR and the option's strike rate. The payment is made, not at option expiration, but at a future date corresponding to the term of the reference rate. For example, an option based on 90-day LIBOR will make a payment 90 days after the expiration date of the option. This payment date often corresponds to the date on which a LIBOR-based borrower would make the next interest payment on a loan.

Example: Computing the payoff for an interest rate option

Assume you bought a 60-day call option on 90-day LIBOR with a notional principal of $1 million and a strike rate of 5 percent. **Compute** the payment that you will receive if 90-day LIBOR is 6 percent at contract expiration, and **determine** when the payment will be received.

Answer:

The interest savings on a $1 million 90-day loan at 5 percent versus 6 percent is:

$$1 \text{ million} \times (0.06 - 0.05)(90 / 360) = \$2{,}500$$

This is the amount that will be paid by the call writer 90 days after expiration.

LOS 72.f: Define interest rate caps and floors.

An **interest rate cap** is a series of interest rate call options, having expiration dates that correspond to the reset dates on a floating rate loan. Caps are often used to protect a floating rate borrower from an increase in interest rates. Caps place a maximum (upper limit) on the interest payments on a floating rate loan.

Caps pay when rates rise above the cap rate. In this regard, a cap can be viewed as a series of interest rate call options with strike rates equal to the cap rate. Each option in a cap is called a *caplet*.

An **interest rate floor** is a series of interest rate put options, having expiration dates that correspond to the reset dates on a floating rate loan. Floors are often used to protect a floating rate lender from a decline in interest rates. Floors place a minimum (lower limit) on the interest payments that are received from a floating rate loan.

An interest rate floor on a loan operates just the opposite of a cap. The floor rate is a minimum rate on the payments on a floating rate loan.

Floors pay when rates fall below the floor rate. In this regard, a floor can be viewed as a series of interest rate put options with strike rates equal to the floor rate. Each option in a floor is called a *floorlet*.

An **interest rate collar** combines a cap and a floor. A borrower with a floating rate loan may *buy* a cap for protection against rates above the cap and *sell* a floor in order to defray some of the cost of the cap.

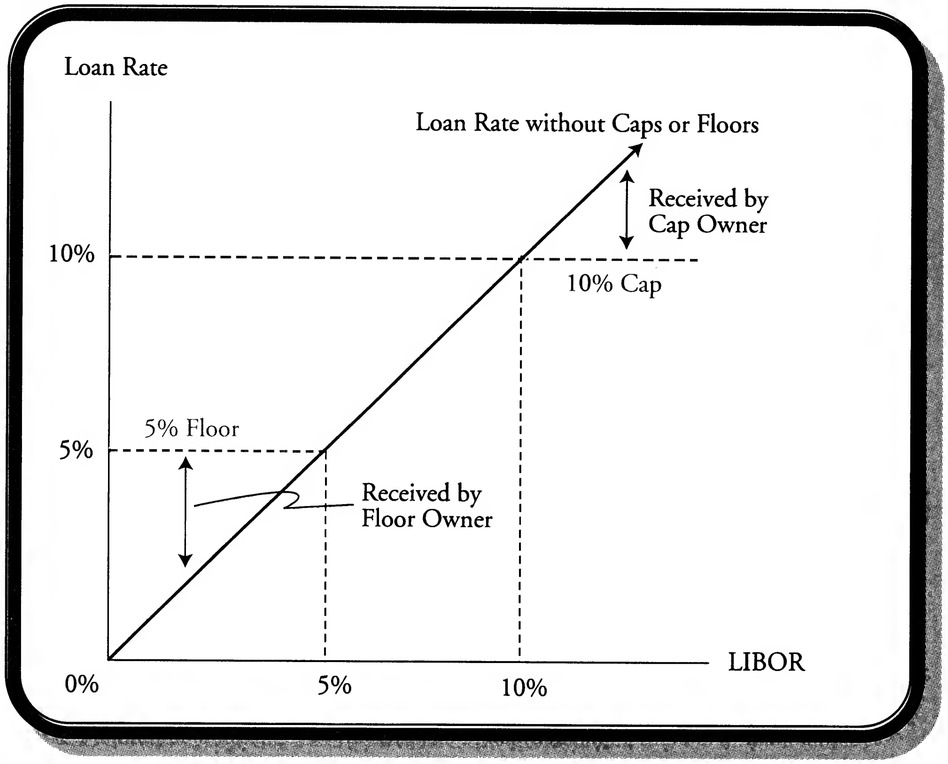
Let's review the information in Figure 3, which illustrates the payments from a cap and a floor. On each reset date of a floating rate loan, the interest for the next period (e.g., 90 days) is determined on the basis of some reference rate. Here, we assume that LIBOR is the reference rate and that we have quarterly payment dates on the loan.

The figure shows the effect of a cap that is set at 10 percent. In the event that LIBOR rises above 10 percent, the cap will make a payment to the cap buyer to offset any interest expense in excess of an annual rate of 10 percent. A cap may be structured to cover a certain number of periods or for the entire life of a loan. The cap will make a payment at any future interest payment due date whenever the reference rate (LIBOR in our example) exceeds the cap rate. As indicated in the figure, the cap's payment is based on the difference between the reference rate and the cap rate. The amount of the payment will equal the notional amount specified in the cap contract times the difference between the cap rate and the reference rate. When used to hedge a loan, the notional amount is usually equal to the loan amount.

Figure 3 also illustrates a floor of 5 percent for our LIBOR-based loan. For any payment where the reference rate on the loan falls below 5 percent, there is an additional payment required by the floor to bring the total payment to 5 percent (1.25 percent quarterly on a 90-day LIBOR-based loan). Note that the issuer of a floating rate note with both a cap and a floor (a collar) is long a cap and *short* (has 'sold') a floor. The issuer receives a payment

when rates are above the cap, and makes an additional payment when rates are below the floor (compared to just paying the reference rate).

Figure 3: Interest Rate Caps and Floors



LOS 72.g: Identify the minimum and maximum values of European options and American options.

The following is some option terminology that we will use when addressing this LOS:

S_t = the price of the underlying stock at time t
X = the exercise price of the option
T = the time to expiration
c_t = the price of a European call at time *t* at any time *t* prior to expiration at time t = T
C_t = the price of an American call at time *t* at any time *t* prior to expiration at time t = T
p_t = the price of a European put at time *t* at any time *t* prior to expiration at time t = T
P_t = the price of an American put at time *t* at any time *t* prior to expiration at time t = T
RFR = the risk-free rate

Professor's Note: Please notice that lowercase letters are used to represent European-style options.

Lower bound. Theoretically, no option will sell for less than its intrinsic value and no option can take on a negative value. This means that the lower bound for any option is zero. *The lower bound of zero applies to both American and European options.*

Upper bound for call options. The maximum value of either an American or a European *call* option at any time *t* is the time-*t* share price of the underlying stock. This makes sense because no one would pay a price for the right to buy an asset that exceeded the asset's value. It would be cheaper to simply buy the underlying asset. At time t = 0, the upper boundary condition can be expressed respectively for American and European call options as:

$$C_0 \leq S_0 \text{ and } c_0 \leq S_0$$

Upper bound for put options. The price for an American put option cannot be more than its strike price. This is the exercise value in the event the underlying stock price goes to zero. However, since European puts cannot be exercised prior to expiration, the maximum value is the present value of the exercise price discounted at the risk-free rate. Even if the stock price goes to zero, and is expected to stay at zero, the intrinsic value, *X*, will not be received until the expiration date. At time t = 0, the upper boundary condition can be expressed for American and European put options, respectively, as:

$$P_0 \leq X \text{ and } p_0 \leq \frac{X}{(1+\text{RFR})^T}$$

The minimum and maximum boundary conditions for the various types of options at any time *t* are summarized in Figure 4.

Figure 4: Option Value Limits

Option	*Minimum Value*	*Maximum Value*
European call	$c_t \geq 0$	$c_t \leq S_t$
American call	$C_t \geq 0$	$C_t \leq S_t$
European put	$p_t \geq 0$	$p_t \leq X/(1 + \text{RFR})^{(T-t)}$
American put	$P_t \geq 0$	$P_t \leq X$

Professor's Note: The values in the table are the theoretical limits on the value of options. In the next section, we will establish more restrictive limits for option prices. (See Exam Flashback #2.)

LOS 72.h: Explain how the lower bounds of European calls and puts are determined by constructing portfolio combinations that prevent arbitrage, and calculate an option's lower bound.

Professor's Note: The option boundary conditions that we discuss below will be important when you study option pricing models. For now, if you follow the logic leading up to the results presented in Figure 5, you will be prepared to deal with this LOS on the exam.

At this point, we know that for **American-style options**, which can be immediately exercised, the minimum price has to be the option's intrinsic value. For at-the-money and out-of-the money options, this minimum is zero, since options cannot have negative values. For in-the-money American options, the minima are simply the intrinsic values S – X for calls, and X – S for puts. If this were not the case, you could buy the option for less than its intrinsic value and immediately exercise it for a guaranteed profit. So, for American options, we can express the *lower bound on the option price* at any time *t* prior to expiration as:

$$C_t = \text{Max }[0, S_t - X]$$
$$P_t = \text{Max }[0, X - S_t]$$

For European options, however, the minima are not so obvious because these options are not exercisable immediately. To determine the **lower bounds for European options,** we can examine the value of a portfolio in which the option is combined with a long or short position in the stock and a pure discount bond.

For a *European call option,* construct the following portfolio:

- A long at-the-money European call option with exercise price *X*, expiring at time $t = T$.
- A long discount bond priced to yield the risk-free rate that pays *X* at option expiration.
- A short position in one share of the underlying stock priced at $S_0 = X$.

The current value of this portfolio is $c_0 - S_0 + X / (1 + RFR)^T$.

At expiration time, $t = T$, this portfolio will pay $c_T - S_T + X$. That is, we will collect $c_T = \text{Max}[0, S_T - X]$ on the call option, pay S_T to cover our short stock position, and collect *X* from the maturing bond.

- If $S_T \geq X$, the call is in-the-money, and the portfolio will have a zero payoff because the call pays $S_T - X$, the bond pays $+X$, and we pay $-S_T$ to cover our short position. That is, the time $t = T$ payoff is: $S_T - X + X - S_T = 0$.
- If $X > S_T$ the call is out-of-the-money, and the portfolio has a positive payoff equal to $X - S_T$ because the call value, c_T, is zero, we collect *X* on the bond, and pay $-S_T$ to cover the short position. So, the time $t = T$ payoff is: $0 + X - S_T = X - S_T$.

Note that no matter whether the option expires in-the-money, at-the-money, or out-of-the-money, the portfolio value will be equal to or greater than zero. We will never have to make a payment.

To prevent arbitrage, any portfolio that has no possibility of a negative payoff cannot have a negative value. Thus, we can state the value of the portfolio *at time t = 0* as:

$$c_0 - S_0 + X / (1 + RFR)^T \geq 0$$

which allows us to conclude that:

$$c_0 \geq S_0 - X / (1 + RFR)^T$$

Combining this result with the earlier minimum on the call value of zero, we can write:

$$c_0 \geq \text{Max}[0, S_0 - X / (1 + RFR)^T]$$

Note that $X / (1 + RFR)^T$ is the present value of a pure discount bond with a face value of *X*.

Based on these results, we can now state the **lower bound for the price of an American call** as:

$$C_0 \geq \text{Max}[0, S_0 - X / (1 + RFR)^T]$$

How can we say this? This conclusion follows from the following two facts:

- The early exercise feature on an American call makes it worth at least as much as an equivalent European call (i.e., $C_t \geq c_t$).
- The lower bound for the value of a European call is equal to or greater than the theoretical lower bound for an American call:
  $\{$i.e., $\text{Max}[0, S_0 - X /(1 + RFR)^T] \geq \text{Max}[0, S_0 - X]\}$

Professor's Note: Don't get bogged down here. We just use the fact that an American call is worth at least as much as a European call to claim that the lower bound on an American call is at least as much as the lower bound on a European call.

Derive the **minimum value of a European put option** by forming the following portfolio at time t = 0:

- A long at-the-money European put option with exercise price *X*, expiring at t = T.
- A short position on a risk-free bond priced at $X / (1 + RFR)^T$. This is the same as borrowing an amount equal to $X / (1 + RFR)^T$.
- A long position in a share of the underlying stock priced at S_0.

At expiration time t = T, this portfolio will pay $p_T + S_T - X$. That is, we will collect $p_T = Max[0, X - S_T]$ on the put option, receive S_T from the stock, and pay $-X$ on the bond issue (loan).

- If $S_T > X$, the payoff will equal: $p_T + S_T - X = S_T - X$.
- If $S_T \leq X$, the payoff will be zero.

Again, a no-arbitrage argument can be made that the portfolio value must be zero or greater, since there are no negative payoffs to the portfolio.

At time t = 0, this condition can be written as:

$$p_0 + S_0 - X / (1 + RFR)^T \geq 0$$

and rearranged to state the minimum value for a European put option at time t = 0 as:

$$p_0 \geq X / (1 + RFR)^T - S_0$$

We have now established the **minimum bound on the price of a European put option** as:

$$p_0 \geq Max[0, X / (1 + RFR)^T - S_0]$$

Professor's Note: Notice that the lower bound on a European put is below that of an American put option (i.e., $Max[0, X - S_0]$*). This is because, when in-the-money, the American put option can be exercised immediately for a payoff of* $X - S_0$.

Figure 5 summarizes what we now know regarding the boundary prices for American and European options at any time *t* prior to expiration at time t = T.

Figure 5: Lower and Upper Bounds for Options

Option	*Minimum Value*	*Maximum Value*
European call	$c_t \geq Max[0, S_t - X / (1 + RFR)^{T-t}]$	S_t
American call	$C_T \geq Max[0, S_t - X / (1 + RFR)^{T-t}]]$	S_t
European put	$p_t \geq Max[0, X / (1 + RFR)^{T-t} - S_t]$	$X / (1 + RFR)^{T-t}$
American put	$P_t \geq Max[0, X - S_t]$	X

Professor's Note: For the exam, know the price limits in Figure 5. You will not be asked to derive them, but may be expected to use them.

LOS 72.i: Determine the lowest prices of European and American calls and puts based on the rules for minimum values and lower bounds.

Example: Minimum prices for American vs. European puts

Compute the lowest possible price for 4-month American and European 65 puts on a stock that is trading at 63 when the risk-free rate is 5 percent.

Answer:

American put: $P_0 \geq \text{Max}\ [0, X - S_0] = \text{Max}[0,2] = \$2$
European put: $p_0 \geq \text{Max}\ [0, X / (1 + \text{RFR})^T - S_0] = \text{Max}[0, 65 / 1.05^{0.333} - 63] = \$0.95$

Example: Minimum prices for American vs. European calls

Compute the lowest possible price for 3-month American and European 65 calls on a stock that is trading at 68 when the risk-free rate is 5 percent.

Answer:

$C_0 \geq \text{Max}\ [0, S_0 - X / (1 + \text{RFR})^T] = \text{Max}[0, 68 - 65 / 1.05^{0.25}] = \$3.79$
$c_0 \geq \text{Max}\ [0, S_0 - X / (1 + \text{RFR})^T] = \text{Max}[0, 68 - 65 / 1.05^{0.25}] = \$3.79$

LOS 72.j: Describe how a portfolio (combination) of options establishes the relationship between options that differ only by exercise price.

The result we are after here is a simple and somewhat intuitive one. That is, given two puts that are identical in all respects except exercise price, the one with the higher exercise price will have at least as much value as the one with the lower exercise price. This is because the underlying stock can be sold at a higher price. Similarly, given two calls that are identical in every respect except exercise price, the one with the lower exercise price will have at least as much value as the one with the higher exercise price. This is because the underlying stock can be purchased at a lower price. The method here, for both puts and calls, is to combine two options with different exercise prices into a portfolio and examine the portfolio payoffs at expiration for the three possible stock price ranges. We use the fact that a portfolio with no possibility of a negative payoff cannot have a negative value to establish the pricing relations for options with differing times to expiration.

For $X_1 < X_2$, consider a portfolio at time t that holds the following positions:

$c_t(X_1)$ = a long call with an exercise price of X_1
$c_t(X_2)$ = a short call with an exercise price of X_2

The three expiration date (t = T) conditions and payoffs that need to be considered here are summarized in Figure 6.

Figure 6: Exercise Price vs. Call Price

Expiration Date Condition	*Option Value*	*Portfolio Payoff*
$S_T \le X_1$	$c_T(X_1) = c_T(X_2) = 0$	0
$X_1 < S_T < X_2$	$c_T(X_1) = S_T - X_1$ $c_T(X_2) = 0$	$S_T - X_1 > 0$
$X_2 \le S_T$	$c_T(X_1) = S_T - X_1$ $c_T(X_2) = S_T - X_2$	$(S_T - X_1) - (S_T - X_2)$ $= X_2 - X_1 > 0$

With no negative payoffs at expiration, the current portfolio of $c_0(X_1) - c_0(X_2)$ must have a value greater than or equal to zero, and we have proven that $c_0(X_1) \ge c_0(X_2)$.

Similarly, consider a portfolio short a put with exercise price X_1 and long a put with exercise price X_2, where $X_1 < X_2$. The expiration date payoffs that we need to consider are summarized in Figure 7.

Figure 7: Exercise Price vs. Put Price

Expiration Date Condition	*Option Value*	*Portfolio Payoff*
$S_T \ge X_2$	$p_T(X_1) = p_T(X_2) = 0$	0
$X_2 > S_T > X_1$	$p_T(X_1) = 0$ $p_T(X_2) = X_2 - S_T$	$X_2 - S_T > 0$
$X_1 \ge S_T$	$p_T(X_1) = X_1 - S_T$ $p_T(X_2) = X_2 - S_T$	$(X_2 - S_T) - (X_1 - S_T)$ $= X_2 - X_1 > 0$

Here again, with no negative payoffs at expiration, the current portfolio of $p_0(X_2) - p_0(X_1)$ must have a value greater than or equal to zero, which proves that $p_0(X_2) \ge p_0(X_1)$.

In summary, we have shown that, all else being equal:

- Call prices are inversely related to exercise prices.
- Put prices are directly related to exercise price.

(See Exam Flashback #3.)

LOS 72.k: Explain how option prices are affected by the time to expiration of the option.

In general, a **longer time to expiration** will increase an option's value. For far out-of-the-money options, the extra time may have no effect, but we can say the longer-term option will be no less valuable that the shorter-term option.

The case that doesn't fit this pattern is the European put. Recall that the minimum value of an in-the-money European put at any time *t* prior to expiration is $X / (1 + RFR)^{T-t} - S_t$. While longer time to expiration increases option value through increased volatility, it decreases the present value of any option payoff at expiration. For this reason, we cannot state positively that the value of a longer European put will be greater than the value of a shorter-term put.

If volatility is high and the discount rate low, the extra time value will be the dominant factor and the longer-term put will be more valuable. Low volatility and high interest rates have the opposite effect and the value of a longer-term in-the-money put option can be less than the value of a shorter-term put option.

LOS 72.l: Illustrate put-call parity for European options, given the payoffs on a fiduciary call and a protective put.

Our derivation of **put-call parity** is based on the payoffs of two portfolio combinations, of a fiduciary call and a protective put.

A *fiduciary call* is a combination of a pure-discount, riskless bond that pays X at maturity and a call with exercise price X. The payoff for a fiduciary call at expiration is X when the call is out-of-the-money, and $X + (S - X) = S$ when the call is in-the-money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is $(X - S) + S = X$ when the put is in-the-money, and S when the put is out-of-the-money.

Professor's Note: When working with put-call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to X.

When the put is in-the-money, the call is out-of-the-money, and both portfolios pay X at expiration.

Similarly, when the put is out-of-the-money and the call is in-the-money, both portfolios pay S at expiration.

Put-call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + X / (1 + RFR)^T = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$S = c - p + X / (1 + RFR)^T$$
$$p = c - S + X / (1 + RFR)^T$$
$$c = S + p - X / (1 + RFR)^T$$
$$X / (1 + RFR)^T = S + p - c$$

The single securities on the left-hand side of the equations all have exactly the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the "synthetic" equivalents of the securities on the left. Note that the options must be European-style and the puts and calls must have the same exercise prices for these relations to hold.

For example, to synthetically produce the payoff for a long position in a share of stock, you use the relationship:

$$S = c - p + X / (1 + RFR)^T$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put-call parity relationship can be constructed in a similar manner.

Professor's Note: After expressing the put-call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (– sign) in the respective securities.

Example: Call option valuation using put-call parity

Suppose that the current stock price is $52 and the risk-free rate is 5 percent. You have found a quote for a 3-month put option with an exercise price of $50. The put price is $1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, $50 call. **Estimate** the price of the 3-month call option.

Answer:

Rearranging put-call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - \text{present value}(X)$$
$$\text{call} = \$1.50 + \$52 - \frac{\$50}{1.05^{0.25}} = \$4.11$$

This means that if a 3-month, $50 call is available, it should be priced at $4.11 per share.

LOS 72.m: Explain the relationship between American options and European options in terms of the lower bounds on option prices and the possibility of early exercise.

Earlier we established that American calls on non-dividend-paying stocks are worth at least as much as European calls, which means that the lower bound on the price of both types of options is Max $[0, S_t - X / (1 + RFR)^{T-t}]$. If exercised, an American call will pay $S_t - X$, which is less than its minimum value of $S_t - X / (1 + RFR)^{T-t}$. Thus, *there is no reason for early exercise of an American call option* on stocks with no dividends.

For American call options on dividend-paying stocks, the argument presented above against early exercise does not necessarily apply. Keeping in mind that options are not typically adjusted for dividends, it may be advantageous to exercise an American call prior to the stock's ex-dividend date, particularly if the dividend is expected to significantly decrease the price of the stock.

For American put options, early exercise may be warranted if the company that issued the underlying stock is in bankruptcy so that its stock price is zero. It is better to get *X* now than at expiration. Similarly, a very low stock price might also make an American put "worth more dead than alive."

LOS 72.n: Explain how cash flows on the underlying asset affect put-call parity and the lower bounds of option prices.

If the asset has positive cash flows over the period of the option, the cost of the asset is less by the present value of the cash flows. You can think of buying a stock for S and simultaneously borrowing the present value of the cash flows, PV_{CF}. The cash flow(s) will provide the payoff of the loan(s), and the loan(s) will reduce the net cost of the asset to $S - PV_{CF}$. Therefore, for assets with positive cash flows over the term of the option, we can **substitute this (lower) net cost, $S - PV_{CF}$**, for S in the lower bound conditions and in all the parity relations.

The lower bounds for European options at time t = 0 can be expressed as:

$c_0 \geq \text{Max } [0, S_0 - PV_{CF} - X / (1 + RFR)^T]$, and
$p_0 \geq \text{Max } [0, X / (1 + RFR)^T - (S_0 - PV_{CF})]$

The put-call parity relations can be adjusted to account for asset cash flows in the same manner. That is:

$$(S_0 - PV_{CF}) = C - P + X / (1 + RFR)^T, \text{ and}$$
$$C + X / (1 + RFR)^T = (S_0 - PV_{CF}) + P$$

LOS 72.o: Identify the directional effect of an interest rate change on an option's price.

When **interest rates increase, the value of a call option increases and the value of a put option decreases.**

The no-arbitrage relations for puts and calls make these statements obvious.

$$C = S + P - X / (1 + RFR)^T$$
$$P = C - S + X / (1 + RFR)^T$$

Here we can see that an increase in RFR decreases $X / (1 + RFR)^T$. This will have the effect of increasing the value of the call, and decreasing the value of the put. A decrease in interest rates will decrease the value of a call option and increase the value of a put option.

Professor's Note: Admittedly, this is a partial analysis of these equations, but it does give the right directions for the effects of interest rate changes and will help you if this LOS is tested on the exam.

LOS 72.p: Describe the impact of a change in volatility on an option's price.

Greater volatility in the value of an asset or interest rate underlying an option contract increases the values of both puts and calls (and caps and floors). The reason is that options are 'one-sided'. Since an option's value falls no lower than zero when it expires out of the money, the increased upside potential (with no greater downside risk) from increased volatility, increases the option's value.

KEY CONCEPTS

1. A call-put option buyer has the right to purchase/sell an underlying asset at a specified price for a specified time.
2. A call-put option writer (seller) has an obligation to sell/buy an underlying asset at a specified price for a specified time period.
3. An option's exercise or strike price is the price at which the underlying stock can be bought or sold by exercising the option.
4. An option's premium is the market price of the option.
5. Moneyness for puts and calls is summarized in the following table:

Moneyness	*Call Option*	*Put Option*
In-the-money	$S > X$	$S < X$
At-the-money	$S = X$	$S = X$
Out-of-the-money	$S < X$	$S > X$

6. American options can be exercised at any time up to the option's expiration date. European options can only be exercised at the option's expiration date.
   - American option values will equal or exceed European option values.
   - American options are more common.
   - European options are easier to analyze.

7. The intrinsic value of an option is the payoff from immediate exercise if it is in-the-money, and zero otherwise.
8. Options are available on:
   - Financial securities.
   - Futures contracts.
   - Commodities.
9. An FRA can be replicated with two interest rate options: long a call and short a put.
10. Interest rate option payoffs are the difference between the market and strike rates, times the principal amount. They are paid after option expiration at the end of the interest rate period specified in the contract.
11. Interest rate caps put a maximum (upper limit) on the payments on a floating rate loan and are equivalent to a series of long interest rate calls from the borrower's perspective.
12. Interest rate floors put a minimum (lower limit) on the payments on a floating rate loan and are equivalent to a series of short interest rate puts from the borrower's perspective.
13. Minimum and maximum option values:

Option	*Minimum Value*	*Maximum Value*
European call	$c_t \geq \text{Max}[0, S_t - X / (1 + RFR)^{T-t}]$	S_t
American call	$C_t \geq \text{Max}\ [0, S_t - X / (1 + RFR)^{T-t}]$	S_t
European put	$p_t \geq \text{Max}[0, X / (1 + RFR)^{T-t} - S_t]$	$X / (1 + RFR)^{T-t}$
American put	$P_t \geq \text{Max}[0, X - S_t]$	X

14. When $X_2 > X_1$, $C_0(X_1) \geq C_0(X_2)$. Calls with lower exercise prices are worth at least as much as (otherwise identical) calls with higher exercise prices (and probably more).
15. When $X_1 > X_2$, $P_0(X_1) \geq P_0(X_2)$. Puts with higher exercise prices are worth at least as much as (otherwise identical) puts with lower exercise prices (and probably more).
16. With two exceptions, otherwise identical options are worth more when there is more time to expiration.
   - Far out-of-the-money options with different expiration dates may be equal in price.
   - With European puts, longer time to expiration may decrease an option's value when the asset price is very low.
17. A fiduciary call and a protective put have the same payoffs at expiration, so $C + X / (1 + RFR)^T = S + P$.
18. For stocks without dividends, an American call will not be exercised early, but early exercise of puts is sometimes advantageous.
19. When the underlying asset has positive cash flows, the minima, maxima, and put-call parity relations are adjusted by subtracting the present value of the cash flows from S.
20. An increase in the risk-free rate will increase call values and decrease put values.
21. Increased volatility of the underlying asset increases both put values and call values.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #45 from '92 actual exam.

An American option is more valuable than a European option on the same dividend paying stock with the same terms because the:
A. European option contract is not adjusted for stock splits and stock dividends.
B. American option can be exercised from date of purchase until expiration, but the European option can be exercised only at expiration.
C. European option does not conform to the Black-Scholes model and is often mispriced.
D. American options are traded on U.S. exchanges, which offer much more volume and liquidity.

Exam Flashback # 2

Source: Question #50 from '91 actual exam.

If a stock is selling for $25, the exercise price of a put option on that stock is $20, and the time to expiration of the option is 90 days, the minimum and maximum prices for the put today are:
A. $0 and $5.
B. $0 and $20.
C. $5 and $20.
D. $5 and $25.

Exam Flashback # 3

Source: Question #88 from '97–'98 sample exams.

Which of the following statements describing options is **FALSE**?
A. A put option gives its holder the right to sell an asset for a specified price on or before the option's expiration date.
B. A call option will be exercised only if the market value of the underlying asset is more than the exercise price.
C. A put option's profit increases when the value of the underlying asset increases.
D. A put option will be exercised only if the market value of the underlying asset is less than the exercise price.

CONCEPT CHECKERS: OPTION MARKETS AND CONTRACTS

1. Which of the following statements about moneyness is **FALSE**? When:
   A. $S - X$ is > 0, a call option is in-the-money.
   B. $S - X = 0$, a call option is at-the-money.
   C. $S = X$, a put option is at-the-money.
   D. $S > X$, a put option is in-the-money.

2. Which of the following statements about American and European options is TRUE?
   A. There will always be some price difference between American and European options because of exchange-rate risk.
   B. American options are more widely traded and are thus easier to value.
   C. European options allow for exercise on or before the option expiration date.
   D. Prior to expiration, an American option may have a higher value than an equivalent European option.

3. Which of the following statements about put and call options is **FALSE**?
   A. The price of the option is less volatile than the price of the underlying stock.
   B. Option prices are generally higher the longer the time till the option expires.
   C. For put options, the higher the strike price relative to the stock's underlying price, the more the put is worth.
   D. For call options, the lower the strike price relative to the stock's underlying price, the more the call option is worth.

4. Which of the following statements is **TRUE**?
   A. The writer of a put option has the obligation to sell the asset to the holder of the put option.
   B. The holder of a call option has the obligation to sell to the option writer should the stock's price rise above the strike price.
   C. The holder of a call option has the obligation to buy from the option writer should the stock's price rise above the strike price.
   D. The holder of a put option has the right to sell to the writer of the option.

5. A *decrease* in the market rate of interest will:
   A. increase put and call prices.
   B. decrease put and call prices.
   C. decrease put prices and increase call prices.
   D. increase put prices and decrease call prices.

6. A $40 call on a stock trading at $43 is priced at $5. The time value of the option is:
   A. $2.
   B. $3.
   C. $5.
   D. $8.

7. Prior to expiration, an American put option on a stock:
   A. is bounded by $S - X / (1 + RFR)^T$.
   B. will sell for its intrinsic value.
   C. will never sell for less than its intrinsic value.
   D. can never sell for more than its intrinsic value.

8. The owner of a call option on oil futures with a strike price of $28.70:
   A. Must pay the strike price to exercise the option.
   B. Can exercise the option and take delivery of the oil.
   C. Can exercise the option and take a long position in oil futures.
   D. Would never exercise the option when the prevailing oil price is less than the strike price.

9. The lower bound for a European put option is:
   A. $\text{Max}(0, S - X)$.
   B. $\text{Max}(0, X - S)$.
   C. $\text{Max}\,[0, X / (1 + RFR)^T - S]$.
   D. $\text{Max}\,[0, S - X / (1 + RFR)^T]$.

10. The lower bound for an American call option is:
   A. $\text{Max}(0, S - X)$.
   B. $\text{Max}(0, X - S)$.
   C. $\text{Max}\,[0, X /(1 + RFR)^T - S]$.
   D. $\text{Max}\,[0, S - X / (1 + RFR)^T]$.

11. To account for positive cash flows from the underlying asset, we need to adjust the put-call parity formula by:
    A. adding the future value of the cash flows to S.
    B. adding the future value of the cash flows to X.
    C. subtracting the present value of the cash flows from S.
    D. subtracting the future value of the cash flows from X.

12. A forward rate agreement is equivalent to the following interest rate options:
    A. long a call and a put.
    B. short a put and a call.
    C. short a call and long a put.
    D. long a call and short a put.

13. The payoff on an interest rate option:
    A. comes only at exercise.
    B. is periodic, typically every 90 days.
    C. is greater the higher the "strike" rate.
    D. comes some period after option expiration.

14. An interest rate floor on a floating-rate note (from the issuer's perspective) is equivalent to a series of:
    A. long interest rate puts.
    B. long interest rate calls.
    C. short interest rate puts.
    D. short interest rate calls.

15. Which of the following relations is **FALSE**?
    A. $S = C - P + X / (1 + RFR)^T$.
    B. $P = C - S + X / (1 + RFR)^T$.
    C. $C = S - P + X / (1 + RFR)^T$.
    D. $X / (1 + RFR)^T - P = S - C$.

16. A stock is selling at $40, a 3-month put at $50 is selling for $11, a 3-month call at $50 is selling for $1, and the risk-free rate is 6 percent. How much, if anything, can be made on an arbitrage?
    A. $0 (no arbitrage).
    B. $0.28.
    C. $0.72.
    D. $2.83.

17. Which of the following will *increase* the value of a put option?
    A. An increase in R_f.
    B. An increase in volatility.
    C. A decrease in the exercise price.
    D. A decrease in time to expiration.

Answers – Exam Flashbacks

1. **B** Investors may be willing to pay more for the right to exercise an American option prior to its expiration.

2. **B** Minimum = \$0, maximum = \$20

   Professor's Note: You know that the minimum value is zero, so this narrows the answer down to two choices. You also know that the maximum put value is the exercise price for American puts, and the discounted value of the exercise price for European puts, which will be less than the exercise price. This leaves choice B.

3. **C** Puts are more valuable when the asset price falls.

Answers – Concept Checkers: Option Markets and Contracts

1. **D** A put option is out-of-the-money when S > X and in-the-money when S < X. The other statements are true.

2. **D** American and European options both give the holder the right to exercise the option at expiration. An American option also gives the holder the right of early exercise, so American options will be worth more than European options when the right to early exercise is valuable, and they will have equal value when it is not, $C_t \geq c_t$ and $P_t \geq p_t$.

3. **A** Option prices are *more* volatile than the price of the underlying stock. The other statements are true. Options have time value which means prices are higher the longer the time until the option expires; a lower strike price increases the value of a call option; and a higher strike price increases the value of a put option.

4. **D** The writer of the put option has the obligation to buy, and the holder of the call option has the right, but not the obligation to buy. Stating that the holder of a put option has the right to sell to the writer of the option is a true statement.

5. **D** Interest rates are inversely related to put prices and directly related to call prices.

6. **A** The intrinsic value is S – X = \$43 – \$40 = \$3. So, the time value is \$5 – \$3 = \$2.

7. **C** At any time *t*, an American put will never sell below intrinsic value, but may sell for more than that. The lower bound is, $\text{Max}[0, X - S_t]$.

8. **C** A call on a futures gives the holder the right to buy a futures contract upon exercise. It is not the current price of oil that determines whether the option is in-the-money, it is the price on the relevant futures contract (which may be higher).

9. **C** The lower bound for a European put ranges from zero to the present value of the exercise price less the prevailing stock price, where the exercise price is discounted at the risk-free rate.

10. **D** The lower bound for an American call ranges from zero to the prevailing stock price less the present value of the exercise price discounted at the risk-free rate.

11. **C** If the underlying asset used to establish the put-call parity relationship generates a cash flow prior to expiration, the assets value must be reduced by the present value of the cash flow discounted at the risk-free rate.

12. **D** The payoff to a FRA is equivalent to that of a long interest rate call option and a short interest rate put option.

13. **D** The payment on a long put increases as the strike rate increases, but not for calls. There is only one payment and it comes after option expiration by the term of the underlying rate.

14. **C** Short interest rate puts require a payment when the market rate at expiration is below the strike rate, just as lower rates can require a payment from a floor.

15. **C** The put-call parity relationship is $S + P = C + X / (1 + RFR)^T$. All individual securities can be expressed as rearrangements of this basic relationship.

16. **C** A synthetic stock is: $S = C - P + X / (1 + RFR)^T = \$1 - \$11 + 50 / (1.06)^{0.25} = \$39.28$. Since the stock is selling for \$40, you can short a share of stock for \$40 and buy the synthetic for an immediate arbitrage profit of \$0.72.

17. **B** Increased volatility of the underlying asset increases both put values and call values.

The following is a review of the Derivative Investments principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

SWAP MARKETS AND CONTRACTS

Study Session 16

EXAM FOCUS

This topic review introduces swaps. The first thing you must learn is the mechanics of swaps so that you can calculate the payments on any of the types of swaps covered. Beyond that, you should be able to recognize that the cash flows of a swap can be duplicated with capital markets transactions (make a loan, issue a bond) or with other derivatives (forward rate agreements or interest rate options). Common mistakes are: forgetting that the current-period floating rate determines the next payment, forgetting to adjust the interest rates for the payment period, forgetting to add any margin above the floating rate specified in the swap, and forgetting that currency swaps involve an exchange of currencies at the initiation and termination of the swap. Don't do these things.

WARM-UP: SWAPS

Before we get into the details of swaps, a simple introduction may help you as you go through the different types of swaps. You can view swaps as the exchange of one loan for another. If you lend me \$10,000 at a floating rate and I lend you \$10,000 at a fixed rate, we have created a swap. There is no reason for the \$10,000 to actually change hands, the two loans make this pointless. At each payment date I will make a payment to you based on the floating rate and you will make one to me based on the fixed rate. Again, it makes no sense to exchange the full amounts; the one with the larger payment liability will make a payment of the difference to the other. This describes the payments of a fixed-for-floating or "plain vanilla" swap.

A currency swap can be viewed the same way. If I lend you 1,000,000 euros, at the euro rate of interest, and you lend me the equivalent amount of yen at today's exchange rate at the yen rate of interest, we have done a currency swap. We will "swap" back these same amounts of currency at the maturity date of the two loans. In the interim, I borrowed yen so I make yen interest payments, and you borrowed euros and must make interest payments in euros.

For other types of swaps we just need to describe how the payments are calculated on the loans. For an equity swap, I could promise to make quarterly payments on your loan to me equal to the return on a stock index, and you could promise to make fixed-rate (or floating-rate) payments to me. If the stock index goes down, my payments to you are negative (i.e., you make a fixed-rate payment to me *and* a payment equal to the decline in the index over the quarter). If the index went up over the quarter, I would make a payment based on the percentage increase in the index. Again, the payments could be "netted" so that only the difference changes hands.

This intuitive explanation of swaps should make what follows a bit easier. Now let's dive into the mechanics and terminology of swaps. We have to specify exactly how the interest payments will be calculated, how often they are made, how much is to be loaned and how long the loans are for. Swaps are custom instruments and we can specify any terms both of us can agree on.

LOS 73.a: Describe the characteristics of swap contracts and explain how swaps are terminated.

Swaps are agreements to exchange a series of cash flows on periodic *settlement dates* over a certain time period (e.g., quarterly payments over two years). In the simplest type of swap, one party makes *fixed-rate* interest payments on the notional principal specified in the swap in return for *floating-rate* payments from the other party. At each settlement date, the two payments are *netted* so that only one (net) payment is made. The party with the greater liability makes a payment to the other party. The length of the swap is termed the *tenor* of the swap and the contract ends on the termination date. A swap can be decomposed into a series of forward contracts (FRAs) that expire on the settlement dates.

In many respects, swaps are similar to forwards:

- Swaps typically require no payment by either party at initiation.
- Swaps are custom instruments.
- Swaps are not traded in any organized secondary market.
- Swaps are largely unregulated.
- Default risk is an important aspect of the contracts.
- Most participants in the swaps market are large institutions.
- Individuals are rarely swaps market participants.

There are swaps facilitators who bring together parties with needs for the opposite sides of swaps. There are also dealers, large banks and brokerage firms, who act as principals in trades just as they do in forward contracts. It is a large business, the total notional principal of swaps contracts is estimated at over $50 trillion.

How Swaps are Terminated

There are **four ways to terminate a swap** prior to its original termination date.

- *Mutual termination.* A cash payment can be made by one party that is acceptable to the other party. Like forwards, swaps can accumulate value as market prices or interest rates change. If the party that has been disadvantaged by the market movements is willing to make a payment of the swaps value to the counterparty and the counterparty is willing to accept it, they can mutually terminate the swap.

- *Offsetting contract.* Just as with forwards, if the terms of the original counterparty offers for early termination are unacceptable, the alternative is to enter an offsetting swap. If our 5-year quarterly-pay floating swap has two years to go, we can seek a current price on a pay-fixed (receive floating) swap that will provide our floating payments and leave us with a fixed-rate liability.

  Just as with forwards, exiting a swap may involve taking a loss. Consider the case where we receive 3 percent fixed on our original 5-year pay floating swap, but must pay 4 percent fixed on the offsetting swap. We have "locked in" a loss because we must pay 1 percent higher rates on the offsetting swap than we receive on the swap we are offsetting. We must make quarterly payments for the next two years, and receive nothing in return. Exiting a swap through an offsetting swap with other than the original counterparty will also expose the investor to default risk, just as with forwards.

- *Resale.* It is possible to sell the swap to another party, with the permission of the counterparty to the swap. This would be unusual, however, as there is not a functioning secondary market.

- *Swaption.* A swaption is an option to enter into a swap. The option to enter into an offsetting swap provides an option to terminate an existing swap. Consider that, in the case of the previous 5-year pay floating swap, we purchased a 3-year call option on a 2-year pay fixed swap at 3 percent. Exercising this swap would give us the offsetting swap to exit our original swap. The cost for such protection is the swaption premium paid.

LOS 73.b: Define and give examples of currency swaps and calculate and interpret the payments on a currency swap.

In a **currency swap**, one party makes payments denominated in one currency, while the payments from the other party are made in a second currency. Typically, the notional amounts of the contract, expressed in both currencies at the current exchange rate, are exchanged at contract initiation and returned at the contract termination date in the same amounts.

An example of a currency swap is as follows: Party A pays Party B $10 million at contract initiation in return for 9.8 million euros. On each of the settlement dates, Party A, having received euros, makes payments at a 6 percent annualized rate in euros on the 9.8 million euros to Party B. Party B makes payments at an annualized rate of 5 percent on the $10 million to Party A. These settlement payments are both made. They are not netted as they are in a single currency interest rate swap.

As an example of what motivates a currency swap, consider that a U.S firm, Party A, wishes to establish operations in Australia and wants to finance the costs in Australian dollars (AUD). The firm finds, however, that issuing debt in AUD is relatively more expensive than issuing USD-denominated debt, because they are relatively unknown in Australian financial markets. An alternative to issuing AUD-denominated debt is to issue USD debt and enter into a USD/AUD currency swap. Through a swaps facilitator, the U.S. firm finds an Australian firm, Party B, that faces the same situation in reverse. They wish to issue AUD debt and swap into a USD exposure.

There are **four possible types of currency swaps** available.

- Party A pays a fixed rate on AUD received, and Party B pays a fixed rate on USD received.
- Party A pays a floating rate on AUD received, and Party B pays a fixed rate on USD received.
- Party A pays a fixed rate on AUD received, and Party B pays a floating rate on USD received.
- Party A pays a floating rate on AUD received, and Party B pays a floating rate on USD received.

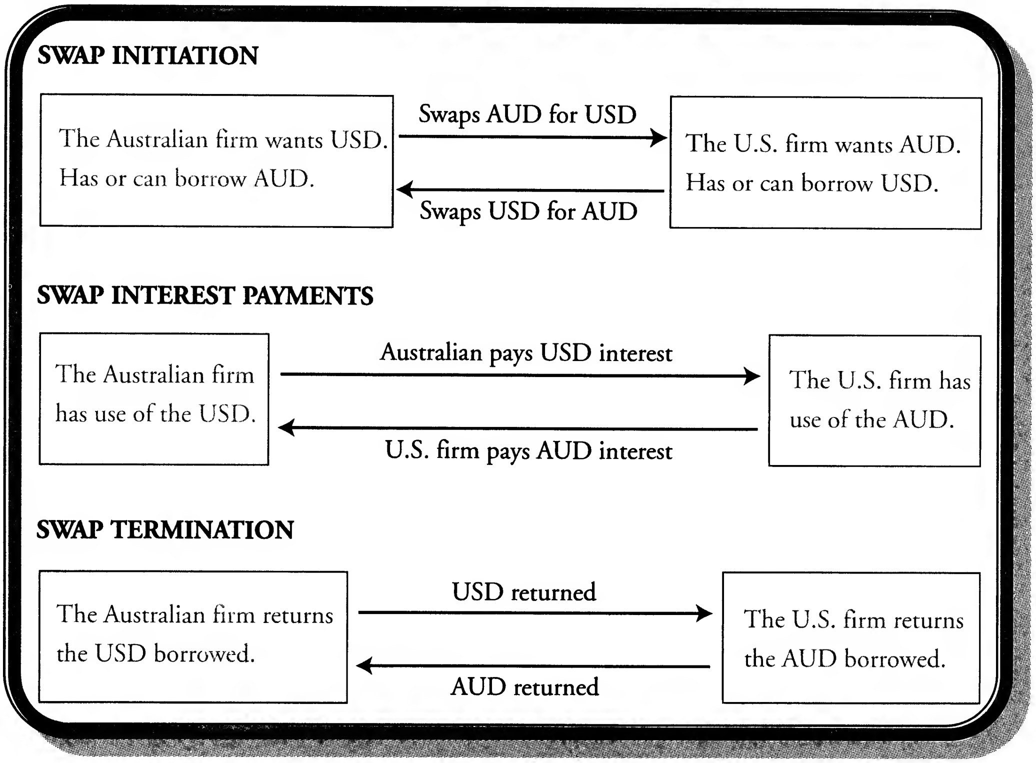
Here are the steps in a fixed-for-fixed currency swap:

The notional principal actually changes hands at the beginning of the swap. Party A gives USD to Party B and gets AUD back. Why? Because the motivation of Party A was to get AUD and the motivation of Party B was to get USD. *Notional principal is swapped at initiation.*

Interest payments are made without netting. Party A, who got AUD, pays the Australian interest rate on the notional amount of AUD to Party B. Party B, who got USD, pays the U.S. interest rate on the notional amount of USD received to Party A. Since the payments are made in different currencies, netting is not a typical practice. *Full interest payments are exchanged at each settlement date, each in a different currency.*

At the termination of the swap agreement (maturity), the counterparties give each other back the exchanged notional amounts. *Notional principal is swapped again at the termination of the agreement.* The cash flows associated with this currency swap are illustrated in Figure 1.

Figure 1: Fixed-for-Fixed Currency Swap



Calculating the Payments on a Currency Swap

Example: Fixed-for-fixed currency swap

BB can borrow in the U.S. for 9 percent, while AA has to pay 10 percent to borrow in the U.S. AA can borrow in Australia for 7 percent, while BB has to pay 8 percent to borrow in Australia. BB will be doing business in Australia and needs AUD, while AA will be doing business in the U.S. and needs USD. The exchange rate is 2AUD/USD. AA needs USD1.0 million and BB needs AUD2.0 million. They decide to borrow the funds locally and swap the borrowed funds, charging each other the rate the other party would have paid had they borrowed in the foreign market. The swap period is for five years. **Calculate** the cash flows for this swap.

Answer:

AA and BB each go to their own domestic bank:

- AA borrows AUD2.0 million, agreeing to pay the bank 7%, or AUD140,000 annually.
- BB borrows USD1.0 million, agreeing to pay the bank 9%, or USD90,000 annually.

AA and BB swap currencies:

- AA gets USD1.0 million, agreeing to pay BB 10% interest in USD annually. BB gets AUD2.0 million, agreeing to pay AA 8% interest in AUD annually.

They pay each other the annual interest:

- AA owes BB USD100,000 in interest to be paid on each settlement date.
- BB owes AA AUD160,000 in interest to be paid on each settlement date.

They each owe their own bank the annual interest payment:

- AA pays the Australian bank AUD140,000 (but gets AUD160,000 from BB, an AUD20,000 gain).
- BB pays the U.S. bank USD90,000 (but gets USD100,000 from AA, a USD10,000 gain).
- They both gain by swapping (AA is ahead AUD20,000, and BB is ahead USD 10,000).

In five years, they reverse the swap. They return the notional principal.

- AA gets AUD2.0 million from BB and then pays back the Australian bank.
- BB gets USD1.0 million from AA and then pays back the U.S. bank.

LOS 73.c: Define and give an example of a plain vanilla interest rate swap and calculate and interpret the payments on an interest rate swap.

The **plain vanilla interest rate swap** involves trading fixed interest rate payments for floating-rate payments.

The party who wants floating-rate interest payments agrees to pay fixed-rate interest and has the *pay-fixed* side of the swap. The counterparty, who receives the fixed payments and agrees to pay variable-rate interest, has the *pay-floating* side of the swap and is called the *floating-rate payer.*

The floating rate quoted is *generally the London Interbank Offered Rate (LIBOR), flat* or plus a spread.

Let's look at the cash flows that occur in a *plain vanilla interest rate swap.*

- Since the notional principal swapped is the same for both counterparties and is in the same currency units, there is no need to actually exchange the cash. *Notional principal is generally not swapped* in single currency swaps.
- The determination of the variable rate is at the beginning of the settlement period, and the cash interest payment is made at the end of the settlement period. Since the interest payments are in the same currency, there is no need for both counterparties to actually transfer the cash. The difference between the fixed-rate payment and the variable-rate payment is calculated and paid to the appropriate counterparty. *Net interest is paid by the one who owes it.*
- At the conclusion of the swap, since the notional principal was not swapped, there is no transfer of funds.

You should note that swaps are a zero-sum game. What one party gains, the other party loses.

The net formula for the *fixed-rate payer,* based on a 360-day year and a floating rate of LIBOR is:

$$(\text{net fixed-rate payment})_t = (\text{swap fixed rate} - \text{LIBOR}_{t-1})\left(\frac{\text{number of days}}{360}\right)(\text{notional principal})$$

If this number is positive, the fixed-rate payer *owes* a net payment to the floating-rate party. If this number is negative, then the fixed-rate payer *receives* a net flow from the floating-rate payer.

Professor's Note: For the exam, remember that with plain vanilla swaps one party pays fixed and the other pays a floating rate. Sometimes swap payments are based on a 365-day year. For example, the swap will specify whether 90/360 or 90/365 should be used to calculate a quarterly swap payment. Remember, these are custom instruments.

Example: Interest rate risk

Consider a bank. Its deposits represent liabilities and are most likely short-term in nature. In other words, deposits represent floating-rate liabilities. The bank assets are primarily loans. Most loans carry fixed rates of interest. The bank assets are fixed-rate and bank liabilities are floating. **Explain** the nature of the interest rate risk that the bank faces and **describe** how an interest rate swap may be used to hedge this risk.

Answer:

The risk the bank faces is that short-term interest rates will rise, causing cash payment on deposits to increase. This would not be a major problem if cash inflows also increase as interest rates rise, but with a fixed-rate loan portfolio they will not. If the bank remains unhedged as interest rates rise, cash outflows rise and bank profits fall.

The bank can hedge this risk by entering into a fixed-for-floating swap as the fixed-rate payer. The floating-rate payments received would offset any increase in the floating-rate payments on deposits. Note that if rates fall, the bank's costs do not. They still pay fixed for the term of the swap and receive (lower) floating-rate payments that correspond to their lower costs on deposits.

Calculating the Payments on an Interest Rate Swap

Example: Calculating the payments on an interest rate swap

Bank A enters into a $1,000,000 quarterly-pay plain vanilla interest rate swap as the fixed-rate payer at a fixed rate of 6 percent based on a 360-day year. The floating-rate payer agrees to pay 90-day LIBOR plus a 1 percent margin; 90-day LIBOR is currently 4 percent.

90-day LIBOR rates are:	4.5%	90 days from now
	5.0%	180 days from now
	5.5%	270 days from now
	6.0%	360 days from now

Calculate the amounts Bank A pays or receives 90, 270, and 360 days from now.

Answer:

The payment 90 days from now depends on current LIBOR and the fixed rate (don't forget the 1 percent margin).

Fixed-rate payer pays:

$$\left[0.06\left(\frac{90}{360}\right)-(0.04+0.01)\left(\frac{90}{360}\right)\right]\times 1{,}000{,}000=\$2{,}500$$

270 days from now the payment is based on LIBOR 180 days from now, which is 5 percent. Adding the 1 percent margin makes the floating-rate 6 percent, which is equal to the fixed rate, so there is no net 3rd quarterly payment.

The Bank's "payment" 360 days from now is:

$$\left[0.06\left(\frac{90}{360}\right)-(0.055+0.01)\left(\frac{90}{360}\right)\right]\times 1{,}000{,}000=-\$1{,}250$$

Since the floating-rate payment exceeds the fixed-rate payment, Bank A will *receive* $1,250 at the 4th payment date. *(See Exam Flashback #1.)*

LOS 73.d: Define and give examples of equity swaps and calculate and interpret the payments on an equity swap.

In an equity swap, the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment. The return can be the capital appreciation or the total return including dividends on the stock, portfolio, or index.

In order to reduce equity risk, a portfolio manager might enter into a 1-year quarterly pay S&P 500 index swap and agree to receive a fixed rate. The percentage increase in the index each quarter is netted against the fixed rate to determine the payment to be made. If the index return is negative, the fixed-rate payer must also pay the percentage decline in the index to the portfolio manager. Uniquely among swaps, equity swap payments can be floating on both sides and the payments are not known until the end of the quarter. With interest rate swaps, both the fixed and floating payments are known at the beginning of period for which they will be paid.

A swap on a single stock can be motivated by a desire to protect the value of a position over the period of the swap. To protect a large capital gain in a single stock, and to avoid a sale for tax or control reasons, an investor could enter into an equity swap as the equity-returns payer, and receive a fixed rate in return. Any decline in the stock price would be paid to the investor at the settlement dates, plus the fixed-rate payment. If the stock appreciates, the investor must pay the appreciation less the fixed payment.

Calculating the Payments on an Equity Swap

Example: Equity swap payments

Ms. Smith enters into a 2-year $10 million quarterly swap as the fixed payer and will receive the index return on the S&P 500. The fixed rate is 8 percent and the index is currently at 986. At the end of the next three quarters the index level is: 1030, 968, and 989.

Calculate the net payment for each of the next three quarters and **identify** the direction of the payment.

Answer:

The percentage change in the index each quarter, Q, is: Q1 = 4.46%, Q2 = –6.02%, and Q3 = 2.17%. The index return payer (IR) will receive 0.08 / 4 = 2% each quarter and pay the index return, therefore:

Q1: IR payer pays 4.46% – 2.00% = 2.46% or $246,000.
Q2: IR payer receives 6.02% + 2.00% = 8.02% or $802,000.
Q3: IR payer pays 2.17% – 2.00% = 0.17% or $17,000.

KEY CONCEPTS

1. Swaps contracts obligate two parties to make payments based on a notional amount, on periodic settlement dates, for the term of the swap (until termination).
2. Swaps may be terminated via mutual agreement between the swap's counterparties, an offsetting contract, resale, or with a swaption.
3. In a plain vanilla interest rate swap, one party pays a floating rate of interest (times the notional amount) on quarterly settlement dates for the tenor of the swap, to the counterparty, who makes fixed-rate payments in return.
   Payments are netted; only the party with the greater liability makes a payment.
4. The net formula for the fixed-rate payer, based on a 360-day year and a floating rate of LIBOR is:

$$(\text{net fixed rate payment})_t = (\text{swap fixed rate} - \text{LIBOR}_{t-1})\left(\frac{\text{number of days}}{360}\right)(\text{notional principal})$$

5. In an equity swap, the returns payer makes payments based on the return on a stock, portfolio, or index, in exchange for fixed-rate payments. If the stock or index declines, the returns payer receives the fixed rate plus any percentage that the equity declines.
6. Currency swap cash flows:
   - Notional principal (in two different currencies) is exchanged at initiation.
   - Interest payments in two different currencies are exchanged on settlement dates.
   - Notional principal amounts are again exchanged at termination.
7. Swaps contracts are similar to forwards in that:
   - They are customized.
   - They do not trade on a secondary market.
   - Default risk is an important consideration.
   - They have zero value at initiation.
   - They are largely unregulated.

Exam Flashbacks

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #97 from '00–'03 sample exams.

Two parties enter a 3-year, plain vanilla interest rate swap agreement to exchange the LIBOR rate for a 10 percent fixed rate on $10 million. LIBOR is 11 percent now, 12 percent at the end of the first year, and 9 percent at the end of the second year. If payments are in arrears, which of the following characterizes the *net cash flow* to be received by the *fixed-rate payer*?
A. $100,000 at the end of year 2.
B. $100,000 at the end of year 3.
C. $200,000 at the end of year 2.
D. $200,000 at the end of year 3.

Concept Checkers: Swap Markets and Contracts

1. Which of the following statements is **FALSE**?
   A. The traders involved in a swap are called the counterparties.
   B. In a plain vanilla interest rate swap, fixed rates are traded for variable rates.
   C. In an interest rate swap, the notional principal is swapped.
   D. The default problem is the most important limitation to the swap market.

2. Which of the following statements is **FALSE**?
   A. In a currency swap, the notional principal is actually swapped twice, once at the beginning of the swap and again at the termination of the swap.
   B. The time frame of a swap is called its tenor.
   C. Swaps are a zero-sum game.
   D. In a currency swap, only net interest payments are made.

3. Which of the following statements is **NOT** an advantage of swaps? Swaps:
   A. give the traders privacy.
   B. have little or no regulation.
   C. minimize default risk.
   D. have customized contracts.

4. In an equity swap:
   A. settlement is made only at swap termination.
   B. shares are exchanged for the notional principal.
   C. returns on two equities are swapped, at each settlement date.
   D. returns on an index can be swapped for fixed-rate payments.

5. In a plain vanilla interest rate swap:
   A. the notional principal is swapped.
   B. only the net notional principal is swapped.
   C. only the net interest payments are made.
   D. the notional principal is returned at the end of the swap.

6. Which of the following statements is **FALSE**?
   A. Only the net difference between the dollar interest and the foreign interest is exchanged in a currency swap.
   B. The notional principal is swapped at inception and at termination of a currency swap.
   C. In an interest rate swap only the net interest is exchanged.
   D. The swap markets are relatively efficient.

Use the following data to answer Questions 7 through 10.

Consider a 3-year annual currency swap that takes place between a foreign firm (FF) with FC currency units and a United States firm (US) with $ currency units. Firm US is the fixed-rate payer and Firm FF is the floating-rate payer. The fixed interest rate at the initiation of the swap is 7 percent, and 8 percent at the end of the swap. The variable rate is 5 percent currently; 6 percent at the end of year 1; 8 percent at the end of year 2; and 7 percent at the end of year 3. At the beginning of the swap, $1.0 million is exchanged at an exchange rate of FC2.0 = $1.0. At the end of the swap period the exchange rate is FD1.5 = $1.0.

Note: With this currency swap, end-of-period payments are based on beginning-of-period interest rates.

7. At the initiation of the swap, which of the following statements is **TRUE**?
   A. FF gives US $1.0 million.
   B. US gives FF $1.0 million.
   C. US gives FF FC2.0 million.
   D. Notional currency is not exchanged in a currency swap.

8. At the end of year 2:
   A. US pays FC140,000; FF pays $60,000.
   B. US pays FC60,000; FF pays $70,000.
   C. US pays FC70,000; FF pays $80,000.
   D. US pays USD70,000; FF pays FC60,000.

9. At the termination of the swap, Firm FF gives Firm US which of the following notional amounts?
   A. $1 million.
   B. $666,666.
   C. FC2,000,000 units.
   D. FC1,500,000 units.

10. At the end of year 3, Firm FF will pay which of the following total amounts?
   A. $1,080,000.
   B. $1,070,000.
   C. FC2,160,000.
   D. FC2,140,000.

Use the following information to answer Questions 11 through 13.

Lambda Corp. has a floating-rate liability and wants a fixed-rate exposure. They enter into a 2-year quarterly-pay \$4,000,000 fixed-for-floating swap as the fixed-rate payer. The counterparty is Gamma Corp. The fixed rate is 6 percent and the floating rate is 90-day LIBOR + 1%, with both calculated based on a 360-day year. Realizations of LIBOR are:

Annualized LIBOR

- Current 5.0%
- In 1 quarter 5.5%
- In 2 quarters 5.4%
- In 3 quarters 5.8%
- In 4 quarters 6.0%

11. The first swap payment is:
    A. from Gamma to Lambda.
    B. known at the initiation of the swap.
    C. \$5,000.
    D. \$20,000.

12. The second net swap payment is:
    A. \$5,000 from Lambda to Gamma.
    B. \$4,000 from Gamma to Lambda.
    C. \$5,000 from Gamma to Lambda.
    D. \$6,000 from Lambda to Gamma.

13. The fifth net quarterly payment on the swap is:
    A. not known, based on the information given.
    B. zero.
    C. \$10,000.
    D. \$40,000.

ANSWERS – EXAM FLASHBACKS

1. C

Year	Fix pay	Var pay	Net	Dollars
1	10%	11%	1% to fixed	+$100,000
2	10%	12%	2% to fixed	+$200,000
3	10%	9%	1% to variable	–$100,000

ANSWERS – CONCEPT CHECKERS: SWAP MARKETS AND CONTRACTS

1. **C** In an interest rate swap, the notional principal is only used to calculate the interest payments and does not change hands. The notional principal is only exchanged in a currency swap.

2. **D** In a currency swap, payments are not netted because they are made in different currencies. Full interest payments are made, and the notional principal is also exchanged.

3. **C** Swaps do not minimize default risk. Swaps are agreements between two or more parties, and there are no guarantees that one of the parties will not default. Note that swaps do give traders privacy and, being private transactions, have little to no regulation and offer the ability to customize contracts to specific needs.

4. **D** Equity swaps involve one party paying the return or total return on a stock or index periodically in exchange for a fixed return.

5. **C** In a plain vanilla interest rate swap, interest payments are netted. Note that notional principal is not exchanged and is only used as a basis for calculating interest payments.

6. **A** In a currency swap, full interest payments are made, and the notional principal is exchanged. Competition in the market for swaps makes pricing relatively efficient.

7. **B** Because this is a currency swap, we know that the notional principal is exchanged. Because the U.S. firm holds dollars, the U.S. firm will be handing over dollars to the foreign firm.

8. **A** Remember, the currency swap is pay floating on dollars and pay fixed on foreign. Floating at the end of year 1 is 6% of $1.0 million. Since payments are made in arrears, Firm FF pays $60,000 and Firm US pays FC140,000 at the end of year 2.

9. **A** The notional principal is exchanged at termination. Firm FF gives back what it borrowed, $1.0 million, and the terminal exchange rate is not used.

10. **A** Firm FF is the floating-rate dollar payer. FF will pay the return of $1.0 million in principal at the termination of the swap, plus the floating rate payment (in arrears) of 8% × $1.0 million = $80,000. The total payment will be $1,080,000.

11. **B** The first payment is based on the fixed rate and current LIBOR + 1%, which are both 6%. There is no net payment made at the first quarterly payment date and this is known at the initiation of the swap.

12. **C** The second quarter payment is based on the realization of LIBOR at the end of the first quarter, 5.5%. The floating rate is: $(5.5\% + 1\%)\left(\frac{90}{360}\right)4{,}000{,}000 = \$65{,}000$. The fixed rate payment is $60,000, making the net payment $5,000 from Gamma to Lambda.

13. **C** The 5th quarterly floating-rate payment is based on the realization of LIBOR at the end of the 4th quarter, which is 6%. With the 1% margin, the floating rate is 7% compared to 6% fixed, so the net payment is $10,000.

The following is a review of the Derivative Investments principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

Risk Management Applications of Options Strategies

Study Session 16

Exam Focus

The most important aspect of this topic review is the interpretation of option profit diagrams. Payoff diagrams for single put or single call positions were covered in our options review. In this review, we introduce profit diagrams and introduce two option strategies that combine stock with options. In a protective put position, we combine a share of stock and a put. With this strategy, we essentially purchase downside protection for the stock (like insurance). A covered call position consists of buying a share of stock and selling a call on it. This strategy equates to selling the upside potential on the stock in return for the added income from the sale of the call. On the Level 1 CFA® exam you will not be expected to draw payoff diagrams, but you will be expected to interpret them and find the breakeven price, maximum gains and losses, and the gains and losses for any stock price at option expiration.

LOS 74.a: Determine the value at expiration, profit, maximum profit, maximum loss, breakeven underlying price at expiration, and general shape of the graph of the strategies of buying and selling calls and buying and selling puts, and explain each strategy's characteristics.

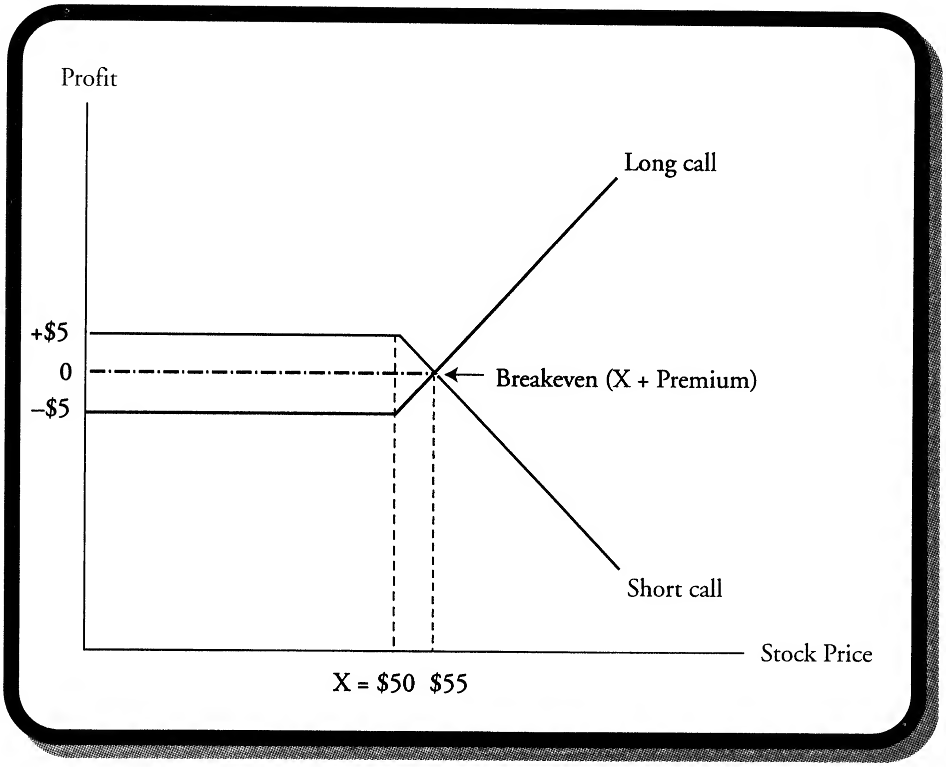
Call option profits and losses. Consider a call option with a premium of \$5 and a strike price of \$50. This means the buyer pays \$5 to the writer. At expiration, if the price of the stock is less than or equal to the \$50 strike price (the option has zero value), the buyer of the option is out \$5, and the writer of the option is ahead \$5. As the stock's price exceeds \$50, the buyer of the option starts to gain (breakeven will come at \$55, when the value of the stock equals the strike price and the option premium). However, as the price of the stock moves upward, the seller of the option starts to lose (negative figures will start at \$55, when the value of the stock equals the strike price and the option premium).

The profit/loss diagram for the buyer (long) and writer (short) of the call option we have been discussing at expiration is presented in Figure 1. This profit/loss diagram illustrates the following:

- The maximum loss for the buyer of a call is the loss of the \$5 premium (at any $S \leq \$50$).
- The breakeven point for the buyer and seller is the strike price plus the premium (at $S = \$55$).
- The profit potential to the buyer of the option is unlimited, and, conversely, the potential loss to the writer of the call option is unlimited.
- The call holder will exercise the option whenever the stock's price exceeds the strike price at the expiration date.
- The greatest profit the writer can make is the \$5 premium (at any $S \leq \$50$).
- The sum of the profits between the buyer and seller of the call option is always zero; thus, options trading is a *zero-sum game*. There are no net profits or losses in the market. The long profits = the short losses.

Professor's Note: Please notice that option profit diagrams show the gain or loss to the long and/or short option positions. They differ from the payoff diagrams that we used in our options review in that profit diagrams reflect the cost of the option (i.e., the option premium).

Figure 1: Profit/Loss Diagram for a Call Option

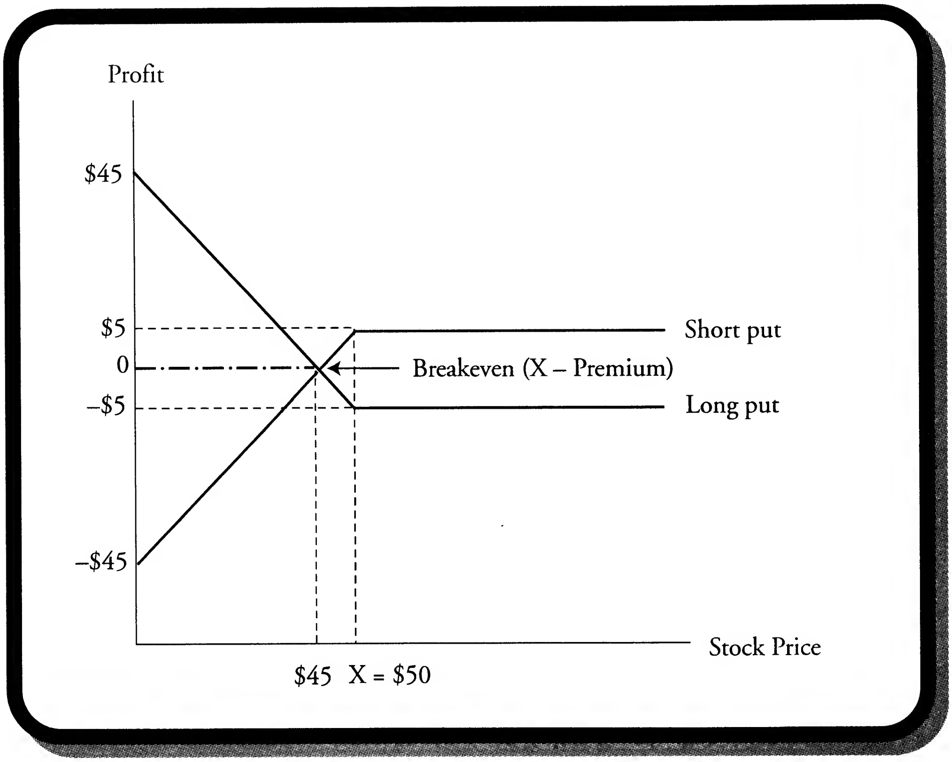


Put option profits and losses. To examine the profits/losses associated with trading put options, consider a put option with a $5 premium. The buyer pays $5 to the writer. When the price of the stock at expiration is greater than or equal to the $50 strike price, the put has zero value. The buyer of the option has a loss of $5, and the writer of the option has a gain of $5. As the stock's price falls below $50, the buyer of the put option starts to gain (breakeven will come at $45, when the value of the stock equals the strike price less the option premium). However, as the price of the stock moves downward, the seller of the option starts to lose (negative profits will start at $45, when the value of the stock equals the strike price less the option premium).

Figure 2 shows the profit/loss diagram for the buyer (long) and seller (short) of the put option that we have been discussing. This profit/loss diagram illustrates that:

- The maximum loss for the buyer of a put is the loss of the $5 premium (at any S ≥ $50).
- The maximum gain to the buyer of a put is limited to the strike price less the premium ($50 – $5 = $45). The potential loss to the writer of the put is the same amount.
- The breakeven price of a put buyer (seller) is at the strike price minus the option premium ($50 – $5 = $45).
- The greatest profit the writer of a put can make is the $5 premium (S ≥ $50).
- The sum of the profits between the buyer and seller of the put option is always zero. Trading put options is a ***zero-sum game***. In other words: the buyer's profits = the writer's losses.

Figure 2: Profit/Loss Diagram for a Put Option



Example: Option profit calculations

Suppose that both a call option and a put option have been written on a stock with an exercise price of $40. The current stock price is $42, and the call and put premiums are $3 and $0.75, respectively.

Calculate the profit to the long and short positions for both the put and the call with an expiration day stock price of $35 and with a price at expiration of $43.

Answer:

Profit will be computed as ending option valuation – initial option cost.

Stock at $35:

- Long call: $0 – $3 = –$3. The option finished out-of-the-money, so the premium is lost.
- Short call: $3 – $0 = $3. Since the option finished out-of-the-money, the call writer's gain equals the premium.
- Long put: $5 – $0.75 = $4.25. You paid $0.75 for an option that is now worth $5.
- Short put: $0.75 – $5 = –$4.25. You received $0.75 for writing the option but you face a $5 loss because the option is in-the-money.

Stock at $43:

- Long call: –$3 + $3 = $0. You paid $3 for the option, and it is now worth $3. Hence, your net profit is zero.
- Short call: $3 – $3 = $0. You received $3 for writing the option and now face a –$3 valuation for a net profit of zero.
- Long put: –$0.75 – $0 = –$0.75. You paid $0.75 for the put option and the option is now worthless. Your net profit is –$0.75.
- Short put: $0.75 – $0 = $0.75. You received $0.75 for writing the option and keep the premium because the option finished out-of-the-money.

LOS 74.b: Determine the value at expiration, profit, maximum profit, maximum loss, breakeven underlying price at expiration, and general shape of the graph of the covered call strategy and the protective put strategy, and explain each strategy's characteristics.

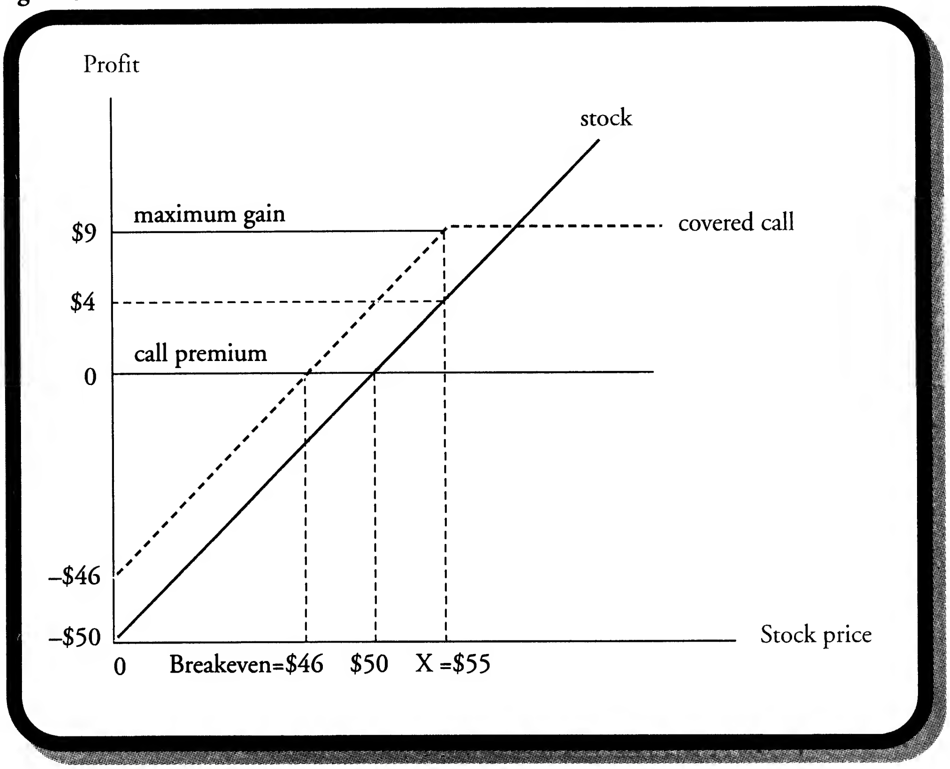
Professor's Note: Whenever we combine options with assets or other options, the net cost of the combined position is simply the sum of the prices paid for the long options/assets minus the proceeds from the option/asset sales (short positions). The profits and losses on a position are simply the value of all the assets/options in the positions at expiration minus the net cost.

In **covered calls**, the term *covered* means that owning the stock covers the inherent obligation assumed in writing the call. Why would you write a covered call? You feel the stock's price will not go up any time soon, and you want to increase your income by collecting the call option premium. To add some insurance that the stock won't get called away, the call writer can write out-of-the-money calls. You should know that this strategy for enhancing one's income is not without risk. *The call writer is trading the stock's upside potential for the call premium.*

Figure 3 illustrates the profit/loss of a covered call position at option expiration date. When the call was written, the stock's price was $50. The call's strike price was $55, and the call premium was $4. The call is out-of-the-money. From Figure 3, we can observe that at expiration:

- If the stock closes below $50, the option will expire worthless, and the option writer's loss is offset by the premium income of $4.
- Breakeven *for the position* is at $46 = $50 – $4. Breakeven price = S_0 – call premium.
- If the stock closes between $50 and $55, the option will expire worthless. Since this option was an out-of-the-money call, the option writer will get any stock appreciation above the original stock price and below the strike price. So the gain (premium plus stock appreciation) will be between $4 and $9.
- If stock closes above $55, the strike price, the writer will get nothing more. The maximum gain is $9 on the covered out-of-the-money call.
- The maximum loss occurs if the stock price goes to zero; the net cost of the position ($46 = $50 stock loss offset by $4 premium income) is the maximum loss.

Figure 3: Covered Call Profit and Loss for S = 50, C = 4, X = 55



The desirability of writing a covered call to enhance income depends upon the chance that the stock price will exceed the exercise price at which the trader writes the call. In this example, the writer of the call thinks the stock's upside potential is less than the buyer expects. The buyer of the call is paying $4 to get any gain above $55, while the seller has traded the upside potential above $55 for a payment of $4.

A **protective put** is an investment management technique designed to protect a stock from a decline in value. It is constructed by buying a stock and put option on that stock.

Look at the profit/loss diagrams in Figure 4. The diagram on the left is the profit from holding the stock. If the stock's value is up, your profit is positive and if the stock's value is down, your profit is negative. Profit equals the end price, S_T, less the initial price S_t. That is, profit = $S_T - S_t$. The diagram on the right side of Figure 4 is the profit graph from holding a long put. If the market is up, you lose your premium payment, and if the market is down, you have a profit.

The value of the put at termination will be $Max[0, X - S_T]$. Your profit will be $Max[0, X - S_T]$ less the price of the put.

Figure 4: Protective Put Components

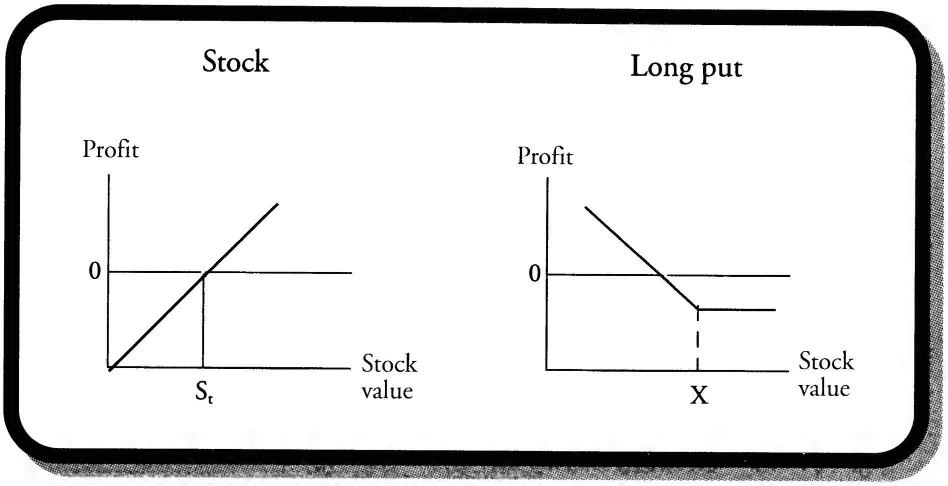
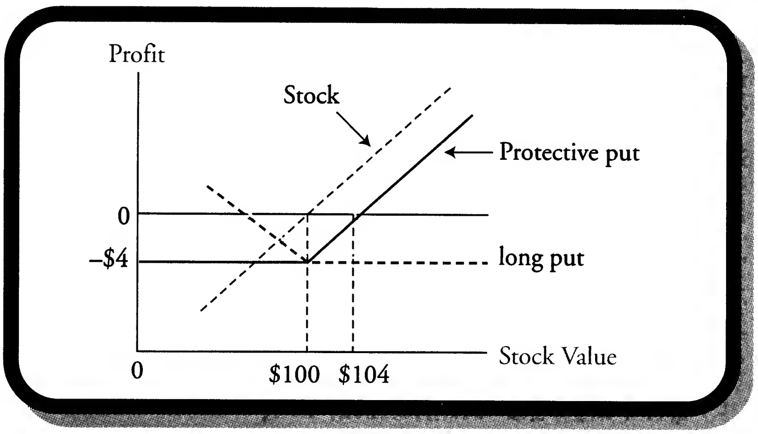


Figure 5 shows the profits from the combination of a long put and a long stock (i.e., a protective put). Here it is assumed that the stock is purchased at \$100 and that a put with a strike price of \$100 is purchased for \$4. Note that the put described in Figure 5 is at-the-money.

Figure 5: Protective Put



What we should observe in Figure 5 is that:

- A protective put cuts your downside losses (maximum loss = \$4) but leaves the upside potential alone (unlimited upside gains).
- Your maximum loss occurs at any price below 100.
- Losses between 0 and \$4 occur for stock prices between \$100 and \$104.
- You will not make a profit until the stock price exceeds \$104 (breakeven).
- Breakeven price = S_0 + premium.

Note that *a protective put (stock plus a put) has the same shape profit diagram as a long call.* It could be replicated with a bond that pays (X – premium) at expiration and a call at X.

KEY CONCEPTS

1. For calls at expiration:
   - The call holder will exercise the option whenever the stock's price exceeds the strike price.
   - The sum of the profits between the buyer and seller of the call option is always zero.

	Call Option	
	Maximum Loss	*Maximum Gain*
Buyer (long)	Premium	Unlimited
Seller (short)	Unlimited	Premium
Breakeven	X + premium	

2. For puts at expiration:
   - The put holder will exercise the option whenever the stock's price is less than the strike price.
   - The sum of the profits between the buyer and seller of the put option is always zero.

	Put Option	
	Maximum Loss	*Maximum Gain*
Buyer (long)	Premium	X – premium
Seller (short)	X – premium	Premium
Breakeven	X – premium	

3. A covered call is the combination of a share of stock and a short (written) call. Profits and losses are measured relative to the net cost of this combination (S_0 – premium).
4. The upside potential on a covered call is limited to ($X - S_0$) + call premium received. The maximum loss is the net cost (S_0 – premium).
5. A protective put strategy consists of buying a share of stock and buying a put. Profits and losses are measured relative to the net cost (S_0 + premium).
6. Maximum gains on a protective put are unlimited, but reduced by the put premium paid. Maximum losses are limited to ($S_0 - X$) + put premium paid.

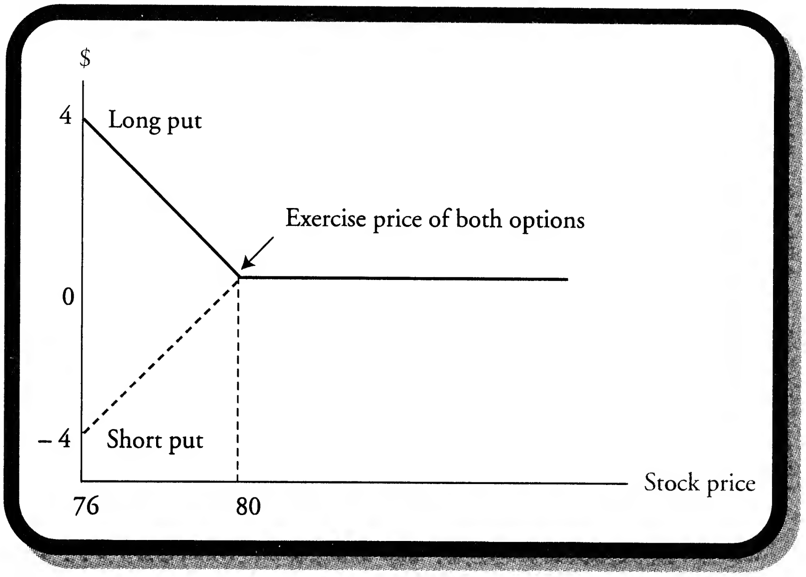
Exam Flashbacks

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #91 from '99-'03 sample exams.

The following diagram shows the value of a put option at expiration:



Ignoring transaction costs, which of the following statements about the value of the put option at expiration is **TRUE**?
A. The value of the short position in the put is $4 if the stock price is $76.
B. The value of the long position in the put is –$4 if the stock price is $76.
C. The long put has value when the stock price is below the $80 exercise price.
D. The value of the short position in the put is zero for stock prices equaling or exceeding $76.

Exam Flashback # 2

Source: Question #93 from '93 and '96 actual exams.

An investor buys two calls and one put on ABC stock, all with a strike price of $45. The calls cost $5 each, and the put costs $4. If the investor closes the position when ABC is priced at $55, the investor's *per share* gain or loss is:
A. $4 loss.
B. $6 gain.
C. $10 gain.
D. $20 gain.

Exam Flashback # 3

Source: Question #99 from '94 and '96 actual exams.

An investor buys a call option with a $25 exercise price priced at $4 and writes a call option with a $40 exercise price priced at $2.50. If the price of the stock increases to $50 at expiration and the options are exercised on the expiration date, the net profit at expiration (ignoring transaction costs) is:
A. $8.50.
B. $13.50.
C. $16.50.
D. $23.50.

Exam Flashback # 4
Source: Question #43 from '92 actual exam.

An at-the-money protective put position (comprised of owning the stock and buying a put):
A. protects against loss at any stock price below the strike price of the put.
B. has limited profit potential when the stock price rises.
C. returns any increase in the stock's value, dollar for dollar, less the cost of the put.
D. provides a pattern of returns similar to a stop loss order at the current stock price.

Concept Checkers: Risk Management Applications of Options Strategies

1. A call option sells for $4 on a $25 stock with a strike price of $30. Which of the following statements is **FALSE**?
   A. At expiration, the buyer of the call will not make a profit unless the stock's price exceeds $30.
   B. At expiration, the writer of the call will only experience a net loss if the price of the stock exceeds $34.
   C. A covered call position at these prices has a maximum gain of $9 and the maximum loss of the stock price less the premium.
   D. A call buyer's maximum loss is $4 and maximum gain is infinite.

2. Assume you bought a put on a stock selling for $60, with a strike price of $55 for a $5 premium. What would be your maximum gain?
   A. $50.
   B. $55.
   C. $60.
   D. $65.

3. Which of the following is the riskiest single-option transaction?
   A. Buying a call.
   B. Writing a call.
   C. Buying a put.
   D. Writing a put.

4. An investor will *likely* exercise a put option when the price of the stock is:
   A. above the strike price.
   B. below the strike price plus the premium.
   C. below the strike price.
   D. equal to the strike price plus a premium.

5. A put with a strike price of $75 sells for $10. Which of the following statements is **FALSE**? The *greatest:*
   A. profit the writer of the put option can make is $10.
   B. profit the buyer of a put option can make is $65.
   C. loss the writer of a put option can have is $75.
   D. loss to the buyer of a put is $10.

6. At expiration, the value of a call option *must equal:*
   A. the larger of the strike price less the stock price or zero.
   B. the stock price minus the strike price, or arbitrage will occur.
   C. zero or arbitrage will occur.
   D. the larger of zero, or the stock's price less the strike price.

7. An investor writes a covered call on a $40 stock with an exercise price of $50 for a premium of $2. The investor's *maximum*:
   A. gain will be $12.
   B. gain will be $2.
   C. loss will be $40.
   D. loss will be unlimited.

8. Which of the following combinations of options and underlying investments have *similarly* shaped profit/loss diagrams?
   A. Covered call and protective put.
   B. Covered call and short stock/long call.
   C. Short put option/long call option and protective put.
   D. Long call option/short put option and long stock position.

Answers – Exam Flashbacks

1. **C** The long put is in-the-money, having positive value when the stock price is below $80.

2. **B** The combined cost of the two options is (2 × $5) + $4 = $14.

   At expiration, the put is worth Max[0, 45 – 55] = 0. Each of the two calls is worth Max[0, S – X] = $10. Thus, the per share gain/loss is –$14 + $0 + (2 × $10) = $6.

3. **B** The net cost is: $4 and received $2.50 = $1.50.
   Gain or loss from the calls: long call $50 – $25 = gain of $25.
   Written call $50 – $40 = $10 loss, net position $25 – $10 = $15.
   Overall gain or loss: gain $15 – cost $1.50 = $13.50 gain.

4. **C** You have no downside risk, so your only loss will be the cost of the put. When the stock price goes up, the put will expire worthless and the stock gives you a dollar for dollar gain. Answer A is not as correct as C. Statement A is true; you are protected against loss at any stock price below the strike price of the put. But the answer didn't mention or consider the cost of the put. The wording of this question was not very clear.

   Professor's Note: Pay attention to the wording of this question. It says the put was at-the-money. This statement eliminates the need to consider what happens to the put's intrinsic value. The put only had time value at the time of its purchase.

Answers – Concept Checkers: Risk Management Applications of Options Strategies

1. **A** The buyer will not have a net profit unless the stock price exceeds $34 (strike price plus the premium). The other statements are true. At $30 the option will be exercised, but the writer will only lose money in a net sense when the stock's price exceeds X + C = $30 + $4. The covered call's maximum gain is $4 premium plus $5 appreciation. The buyer's maximum loss is the premium, and, since the buyer has the rights to all appreciation, the gain is unlimited.

2. **A** This assumes the price of the stock falls to zero and you get to sell for $55. Your profit would be $55 – 5 = $50.

3. **B** When buying either a call or a put, the loss is limited to the amount of the premium. When writing a put, your loss is limited to the strike price of the stock if the stock falls to zero (however, you keep the premium). When writing an uncovered call, the stock could go up infinitely, and you would be forced to buy the stock in the open market and deliver at the strike price—potential losses are unlimited.

4. **C** The owner of a put profits when the stock falls. The put would be exercised when the price of the stock is *below* the strike price. The amount of the premium is used to determine net profits to each party.

5. **C** The greatest loss the put writer can have is the strike price minus the premium received = $65. The other statements are true. The greatest profit the put writer can make is the amount of the premium. The greatest profit for a put buyer occurs if the stock falls to zero and the buyer makes the strike price minus the premium. Since options are a zero-sum game, the maximum profit to the writer of the put must equal the maximum loss to the buyer of the put.

6. **D** At expiration the value of a call must be equal to its intrinsic value, which is Max[0, S – X]. If the value of the stock is less than the strike price, the intrinsic value is zero. If the value of the stock is greater than the strike price, the call is in-the-money and the value of the call is the stock price minus the strike price, or S – X.

7. **A** As soon as the stock rises to the exercise price, the covered call writer will cease to realize a profit because the short call moves into-the-money. Each dollar gain on the stock is then offset with a dollar loss on the short call. Since the option is $10 out-of-the-money, the covered call writer can gain this amount plus the $2 call premium. Thus, the maximum gain is $2 + $10 = $12. However, because the investor owns the stock, he or she could lose $40 if the stock goes to zero, but gain $2 from the call premium. Maximum loss is $38.

8. **D** A long call and a short put will provide a nearly identical payoff as a long stock.

   Professor's Note: The easiest way to see this is to draw the payoff diagram for the combined option positions.

FORMULAS

full price = clean price + accrued interest

$$\text{duration} = -\frac{\text{percentage change in bond price}}{\text{yield change in percent}}$$

value of a callable bond = value of an option-free bond – value of the call

$$\text{TIPS coupon payment} = \text{inflation-adjusted par value} \times \frac{\text{stated coupon rate}}{2}$$

absolute yield spread = yield on the higher-yield bond – yield on the lower-yield bond

$$\text{relative yield spread} = \frac{\text{absolute yield spread}}{\text{yield on the lower-yield bond}} = \frac{\text{higher yield}}{\text{lower yield}} - 1$$

$$\text{yield ratio} = \frac{\text{higher yield}}{\text{lower yield}}$$

$$\text{after-tax yield} = \text{taxable yield} \times (1 - \text{marginal tax rate})$$

$$\text{taxable-equivalent yield} = \frac{\text{tax-free yield}}{(1 - \text{marginal tax rate})}$$

$$\text{bond equivalent yield} = \left[(1 + \text{monthly CFY})^6 - 1\right] \times 2$$

$$S_3 = \left[(1 + {}_1f_0)(1 + {}_1f_1)(1 + {}_1f_2)\right]^{\frac{1}{3}} - 1$$

$$\frac{(1 + S_3)^3}{(1 + S_2)^2} - 1 = {}_1f_2$$

$$\text{effective duration} = \frac{(\text{bond price when yields fall} - \text{bond price when yields rise})}{2 \times (\text{initial price}) \times (\text{change in yield in decimal form})}$$

percentage change in bond price = –effective duration × change in yield in percent

portfolio duration = $w_1D_1 + w_2\,D_2 + \ldots + W_ND_N$

percentage change in price = duration effect + convexity effect

$$= \left\{\left[-\text{duration} \times (\Delta y)\right] + \left[\text{convexity} \times (\Delta y)^2\right]\right\} \times 100$$

duration × 0.0001 × bond value = price value of a basis point

value of a long FRA at settlement: $(\text{notional principal})\dfrac{(\text{floating} - \text{forward})\left(\dfrac{\text{days}}{360}\right)}{1+(\text{floating})\left(\dfrac{\text{days}}{360}\right)}$

intrinsic value of a call: C = Max[0, S – X]

intrinsic value of a put: P = Max[0, X – S]

option value = intrinsic value + time value

lower and upper bounds for options:

Option	*Minimum Value*	*Maximum Value*
European call	$c_t \geq \text{Max}[0, S_t - X / (1 + \text{RFR})^{T-t}]$	S_t
American call	$C_T \geq \text{Max}[0, S_t - X / (1 + \text{RFR})^{T-t}]]$	S_t
European put	$p_t \geq \text{Max}[0, X / (1 + \text{RFR})^{T-t} - S_t]$	$X / (1 + \text{RFR})^{T-t}$
American put	$P_t \geq \text{Max}[0, X - S_t]$	X

put-call parity: $c + X / (1 + \text{RFR})^T = S + p$

put-call parity with asset cash flows: $C + X / (1 + \text{RFR})^T = (S_0 - PV_{CF}) + P$

$$(\text{net fixed-rate payment})_t = (\text{swap fixed rate} - \text{LIBOR}_{t-1})\left(\frac{\text{number of days}}{360}\right)(\text{notional principal})$$

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